pre-IB Mathematics
ANSWERS

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May 25, 2018



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Contents

Contents **3**

1 Numbers **5**
1.1 Primes, factors and divisibility . . . 5
1.2 Fractions and decimals . . . 6
1.3 Subsets of real numbers set . . . 6
1.4 Absolute value . . . 6
1.5 Percentages . . . 6
1.6 Approximations. Decimal places and significant figures. . . . 7
1.7 Exponents and roots . . . 8
1.8 Expansions . . . 9

2 Sets **9**
2.1 Sets and subsets . . . 9
2.2 Venn diagrams . . . 10
2.3 Operations on sets . . . 12
2.4 Chapter review . . . 13

3 Lines and vectors **14**
3.1 Basic concepts . . . 14
3.2 *Slope-intercept* equation of a line . . . 14
3.3 *General* equation of a line . . . 15
3.4 Vectors . . . 15
3.5 Simultaneous equations . . . 17
3.6 Applications of linear equations and vectors . . . 17
3.7 Chapter review . . . 18

4 Functions **19**
4.1 Basic properties . . . 19
4.2 Transformations of graphs of functions . . . 21
4.3 Equations and inequalities . . . 29
4.4 Chapter review . . . 29

5 Quadratics **32**
5.1 Solving quadratic equations . . . 32
Factorisation . . . 32
Completing the square . . . 33
Quadratic formula . . . 33
5.2 Parabola . . . 34
5.3 Applications of quadratics . . . 34
Quadratic inequalities . . . 34
Problems involving quadratics . . . 35
Investigating graphs of rational functions . . . 35

6 Trigonometry **36**
6.1 Degrees and radians . . . 36
6.2 Trigonometric ratios . . . 36
6.3 Trigonometric functions . . . 37
6.4 Trigonometric equations . . . 38
6.5 Trigonometry in geometry . . . 39

6.6 Arcs, sectors, segments . . . 40
6.7 Chapter review . . . 40

7 Statistics **41**
7.1 Types of data . . . 41
7.2 Averages, range, quartiles . . . 41
7.3 Groued data, frequencies . . . 42
7.4 Miscelaneous problems . . . 44

8 Geometry **44**
8.1 Polygons . . . 44
8.2 Circles . . . 44
8.3 Similarity . . . 45
8.4 Solid geometry . . . 45
8.5 Miscellaneous problems . . . 45

9 Numbers II **45**
9.1 Factorials, Pascal's triangle and binomial theorem . . . 45
9.2 Logarithms . . . 46
Algebra of logarithms . . . 46
Logarithmic equations . . . 47
Aplications . . . 47
9.3 Absolute value equations and inequalities . . . 47
9.4 Complex numbers . . . 48
9.5 Mathematical induction . . . 48

10 Quadratics, circles and polynomials **48**
10.1 Vieta's formulae for quadratics . . . 48
10.2 Algebraic fractions . . . 49
10.3 Equation of a circle . . . 49
10.4 Polynomials . . . 50
10.5 Chapter review . . . 51

Chapter 1
Numbers

1.1 Primes, factors and divisibility

Q1. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Q2. (1) 3
(2) co-prime
(3) co-prime
(4) 7
(5) co-prime
(6) 3
(7) co-prime
(8) co-prime
(9) 26
(10) 2
(11) 3
(12) 13
(13) 12
(14) co-prime

Q3. (1) gcd: 6, lcm: 90
(2) gcd: 16, lcm: 96
(3) gcd: 12, lcm: 72
(4) gcd: 18, lcm: 216
(5) gcd: 18, lcm: 630
(6) gcd: 8, lcm: 168
(7) gcd: 7, lcm: 504
(8) gcd: 9, lcm: 648
(9) gcd: 8, lcm: 448
(10) gcd: 99, lcm: 198
(11) gcd: 1, lcm: 4006002
(12) gcd: 1, lcm: 4008003
(13) gcd: 2, lcm: 2002000
(14) gcd: 17, lcm: 595
(15) gcd: 14, lcm: 210
(16) gcd: 7, lcm: 245
(17) gcd: 22, lcm: 1452
(18) gcd: 125, lcm: 5000
(19) gcd: 25, lcm: 10000
(20) gcd: 16, lcm: 640
(21) gcd: 24, lcm: 1728
(22) gcd: 13, lcm: 936

Q4. (1) 2×3^2
(2) $2^3 \times 3$
(3) $2 \times 3 \times 5$
(4) 2^5
(5) $2^2 \times 3^2$
(6) $2 \times 3 \times 7$
(7) $2^4 \times 3$
(8) 2×3^3
(9) $2^3 \times 7$
(10) $3^2 \times 7$
(11) 2^6
(12) $2^3 \times 3^2$
(13) $2^4 \times 5$
(14) 3^4
(15) $2 \times 3^2 \times 5$
(16) $3^2 \times 11$
(17) $3^2 \times 13$
(18) 2^7
(19) $2^6 \times 3$
(20) $2^2 \times 7^2$
(21) $2^3 \times 3^3$
(22) 2×11^2
(23) $2 \times 3 \times 7^2$
(24) $2^4 \times 5^2$
(25) 5^4
(26) $2^3 \times 5^3$
(27) $3 \times 11 \times 61$
(28) $2^2 \times 23$
(29) $2^5 \times 3$
(30) $2^2 \times 5^2$
(31) $2^4 \times 3^2$
(32) 2×3^4
(33) $2^5 \times 7$
(34) $2^4 \times 3 \times 5$
(35) 17^2
(36) $2^2 \times 11^2$
(37) 23^2
(38) $2^6 \times 3^2$
(39) $3^3 \times 5^2$
(40) $2^2 \times 13^2$
(41) $7 \times 11 \times 13$
(42) $3^2 \times 11^2$
(43) $3^2 \times 5^3$
(44) $2^3 \times 17^2$
(45) $2^2 \times 3^2 \times 5^2 \times 7$

Q5. (1) 1
(2) 2
(3) 3
(4) 2
(5) 3
(6) 5
(7) 3
(8) 0
(9) 6
(10) 4
(11) 1
(12) 6
(13) 3
(14) 5
(15) 0
(16) 10

Q6. (1) 3,5
(2) 2,3,4,5,6,10
(3) 3,5
(4) 3,9
(5) none
(6) 2
(7) 2,4
(8) 2,5,10
(9) 5
(10) 2,4
(11) 2,3,4,6,9
(12) 2,3,6

1.2 Fractions and decimals

Q7. (1) recurring
(2) terminating
(3) terminating
(4) terminating
(5) terminating
(6) terminating
(7) recurring
(8) terminating
(9) recurring
(10) terminating
(11) terminating
(12) recurring
(13) recurring
(14) terminating
(15) recurring
(16) recurring
(17) terminating
(18) terminating
(19) terminating
(20) terminating

Q8. (1) $1\frac{6}{25}$
(2) $2\frac{17}{20}$
(3) $\frac{1}{160}$
(4) $3\frac{3}{8}$
(5) $5\frac{5}{8}$
(6) $7\frac{11}{25}$
(7) $12\frac{1}{8}$
(8) $75\frac{7}{8}$
(9) $2\frac{4}{9}$
(10) $3\frac{4}{33}$
(11) $5\frac{5}{9}$
(12) $7\frac{19}{33}$
(13) $14\frac{14}{99}$
(14) $\frac{107}{333}$
(15) $4\frac{202}{333}$
(16) $5\frac{68}{333}$
(17) $\frac{23}{45}$
(18) $1\frac{19}{90}$
(19) $3\frac{19}{30}$
(20) $5\frac{173}{330}$
(21) $6\frac{1}{66}$
(22) $1\frac{523}{1665}$
(23) $7\frac{817}{3330}$
(24) $9\frac{1}{1665}$

Q9. (1) 0.625
(2) 0.1875
(3) 0.875
(4) 0.275
(5) 0.024
(6) $0.\dot{7}$
(7) $0.\dot{2}$
(8) $0.\dot{1}\dot{8}$
(9) $0.\dot{2}\dot{7}$
(10) $0.\dot{2}85714\dot{2}$
(11) $0.8\dot{3}$
(12) $0.2\dot{7}$
(13) $0.41\dot{6}$
(14) $0.1\dot{3}\dot{6}$
(15) $0.\dot{0}\dot{6}$
(16) $0.\dot{1}3\dot{5}$

1.3 Subsets of real numbers set

Q10. (1) $\mathbb{R} \setminus \mathbb{Q}$
(2) $\mathbb{R} \setminus \mathbb{Q}$
(3) $\mathbb{Q}$, $\mathbb{Z}$, $\mathbb{N}$
(4) $\mathbb{Q}$
(5) $\mathbb{Q}$
(6) $\mathbb{Q}$
(7) $\mathbb{Q}$
(8) $\mathbb{Q}$
(9) $\mathbb{Q}$, $\mathbb{Z}$, $\mathbb{N}$
(10) $\mathbb{R} \setminus \mathbb{Q}$
(11) $\mathbb{R} \setminus \mathbb{Q}$
(12) $\mathbb{R} \setminus \mathbb{Q}$

1.4 Absolute value

Q11. (1) 7.2
(2) 3.4
(3) $3.4 - \pi$
(4) $\sqrt{5} - 2$
(5) $\frac{3}{3-2\sqrt{2}}$
(6) $10 - \pi^2$
(7) $2\sqrt{3} - 3$
(8) 16
(9) 8
(10) 0
(11) $3\sqrt{2} - 4$
(12) $2\sqrt{7} - 5$
(13) $5\sqrt{2} - 7$
(14) $\sqrt[3]{9} - 2$
(15) $5 - 2\sqrt{6}$
(16) $2\sqrt{10} - 6$
(17) $\pi - 3$
(18) 13
(19) $7 - 4\sqrt{3}$
(20) $10 - 3\sqrt{11}$

Q12. (1) $4.5; 0.5$
(2) $\frac{1}{3}; 1$
(3) $\frac{1}{4}; -1\frac{1}{4}$
(4) $-7; -3$
(5) $2\frac{1}{3}$
(6) $1\frac{3}{4}; -1\frac{1}{4}$
(7) $-\frac{3}{5}; 1\frac{4}{5}$
(8) no solution
(9) $-1; -1\frac{2}{3}$
(10) $-0.5; 5.5$
(11) $\frac{1}{11}; \frac{5}{11}$
(12) no solution

Q13. (1) $-1; -5$
(2) $-1; -3\frac{2}{3}$
(3) $1\frac{3}{4}; 1\frac{1}{4}$
(4) $3\frac{4}{5}; 2\frac{1}{5}$
(5) $1\frac{1}{3}; -\frac{1}{3}$
(6) $-\frac{2}{5}; -3\frac{3}{5}$
(7) no solution
(8) $0; -\frac{6}{7}$
(9) $\frac{7}{9}; \frac{5}{9}$
(10) $\frac{11}{23}; \frac{12}{23}$
(11) no solution
(12) no solution

1.5 Percentages

Q14. (1) 48
(2) 67
(3) 42
(4) 68
(5) 65

Q15. (1) 6%
(2) 25%
(3) 18%
(4) 14.2%($14\frac{1}{6}$%)
(5) 225%
(6) 240%

Q16. (1) 65
(2) 85
(3) 32
(4) 480
(5) 25

Q17. (1) 16%
(2) 16.7%
(3) 16%
(4) 16.7%
(5) 25%
(6) 20%

Q18. (1) 33.3%
(2) 7.2
(3) 237.5%
(4) 15.2
(5) 77.8%
(6) 22.4
(7) 22.4

Q19. (1) 28.9
(2) 0.65
(3) 65
(4) 14%
(5) 12%

Q20. (1) 18%
(2) 12.5%
(3) 82%
(4) 84%
(5) 76%
(6) 34%
(7) 42%
(8) 22.4%
(9) 456
(10) 255
(11) 70
(12) 85
(13) 65
(14) 165
(15) 65
(16) 11.9
(17) 44.1
(18) 143
(19) 25%
(20) 20%

Q21. 45

Q22. 14%

Q23. 550

Q24. 528 zł

Q25. 187.50 zł

Q26. 1188 zł

Q27. 20%

Q28. 10%

Q29. 10%

Q30. 60 zł

Q31. 3120 zł

Q32. 850 zł

Q33. 5.60 zł

Q34. 25%

Q35. increased by 12.5%

Q36. 301 zł

Q37. 18%

Q38. 378 zł

Q39. on average, 2.04%

Q40. 8.25%

Q41. 7.96%

Q42. 800, final smaller by 14.5%

Q43. 50%

Q44. 300, final smaller by 4%

1.6 Approximations. Decimal places and significant figures.

Q45. (1) 102.44
(2) 2.01
(3) 3.61
(4) 3.90
(5) 14.14
(6) 30.00
(7) 0.01
(8) 0

Q46. (1) 10
(2) 20
(3) 6710
(4) 340
(5) 30
(6) 430
(7) 650
(8) 110

Q47. (1) 2000
(2) 6521000
(3) 12000
(4) 4000
(5) 0
(6) 130000
(7) 43000
(8) 62000

Q48. (1) 20000
(2) 20000
(3) 0.002
(4) 0.0004
(5) 4200000
(6) 25
(7) 0.0021
(8) 0.00025
(9) 25.4
(10) 0.00208
(11) 0.000255
(12) 45600000

Q49. (1) $2335000 \leq a < 2345000$
(2) $932.5 \leq a < 933.5$
(3) $4045000 \leq a < 4055000$
(4) $0.01225 \leq a < 0.01235$
(5) $0.004495 \leq a < 0.004505$
(6) $1995 \leq a < 2005$
(7) $18.95 \leq a < 19.05$
(8) $32450 \leq a < 32550$
(9) $0.09985 \leq a < 0.09995$
(10) $0.002455 \leq a < 0.002465$
(11) $0.4045 \leq a < 0.4055$
(12) $0.06995 \leq a < 0.07005$

1.7 Exponents and roots

Q50. (1) 5 (2) 3 (3) 6 (4) 3 (5) 2 (6) 3 (7) 7 (8) 2 (9) $\frac{2}{3}$ (10) $\frac{5}{2}$ (11) $\frac{4}{3}$ (12) $\frac{7}{2}$ (13) $\frac{3}{2}$ (14) $\frac{5}{3}$ (15) $\frac{7}{3}$ (16) $\frac{3}{2}$ (17) $\frac{5}{2}$ (18) $\frac{4}{3}$ (19) $\frac{3}{2}$

Q51. (1) $2^5 > 5^2$ (2) $2^5 > (-2)^5$ (3) $(-2)^5 < (-2)^4$ (4) $4^0 > 0^4$ (5) $(-2)^5 > (-2)^7$ (6) $(\frac{1}{2})^5 > (\frac{1}{2})^6$ (7) $(-\frac{1}{2})^5 < -(\frac{1}{2})^6$ (8) $(-2)^5 < 2^4$ (9) $(-2)^5 < (-2)^6$ (10) $(-2)^5 > -2^6$ (11) $(-2)^5 = -2^5$ (12) $(-2)^4 > -2^4$

Q52. (1) 3^{14} (2) 3^6 (3) 7^{77} (4) -2^{11} (5) 2^{30} (6) -2^{35} (7) 3^6 (8) 5^7 (9) 3^2 (10) 3^3 (11) 2^{20} (12) 4^{16} (13) 2^{28} (14) 3^{11} (15) 2^{19} (16) 3^8 (17) 9^{-1} (18) $9^0 = 1$ (19) $2^1 = 2$ (20) 2^6 (21) 2^{-35} (22) 5^{-2} (23) 7^4 (24) 3^{-7} (25) 3^{-13} (26) 2^{-19} (27) 5^{-17} (28) 2^{-4}

Q53. $h < f = g < b = d < a = e < c < i = j$

Q54. $f = j < g = i < a = b < c < d = e < h$

Q55. (1) $x^{3.5}$ (2) $a^{\frac{2}{3}}$ (3) a^6 (4) $a^{\frac{8}{3}}$ (5) a^5 (6) x^5 (7) b^5 (8) $c^{\frac{14}{3}}$ (9) y^3 (10) $d^{\frac{9}{4}}$ (11) $s^{\frac{8}{5}}$ (12) t^2 (13) n^3 (14) $n^{\frac{3}{2}}$ (15) a^5 (16) a^3 (17) p^{-3} (18) $s^{-\frac{14}{3}}$ (19) b^{-3} (20) x^{-2} (21) $y^{-5.5}$ (22) $t^{\frac{1}{3}}$ (23) $w^{\frac{1}{3}}$ (24) a^{-7}

Q56. (1) x^{12} (2) $3p^4$ (3) $3x$ (4) $\frac{2}{3}x^2$ (5) $\frac{3}{2}a^2$ (6) $3s^2$ (7) $\frac{3}{2n}$ (8) $6w^3$ (9) $2a$ (10) $\frac{7}{2}p^3$ (11) $\frac{3}{2}a^2$ (12) $\frac{1}{40}s^5$

Q57. (1) 0.00001 (2) $\frac{16}{9}$ (3) $\frac{1}{32}$ (4) 32 (5) $\frac{64}{27}$ (6) $\frac{2}{5}$ (7) $\frac{16}{81}$ (8) $\frac{27}{512}$ (9) 1000000 (10) $\frac{16}{3}$ (11) $\frac{3}{16}$ (12) $\frac{16}{49}$

Q58. (1) 2 (2) 4 (3) $\frac{1}{8}$ (4) 27 (5) $\frac{81}{16}$ (6) $\frac{\sqrt{3}}{3}$ (7) $\frac{9}{4}$ (8) $\frac{1}{4}$ (9) $\frac{1}{5}$ (10) $\frac{2}{3}$ (11) $\frac{2}{5}$ (12) $\frac{81}{16}$ (13) $\frac{1}{3}$ (14) 1024

Q59. (1) $\frac{\sqrt{2}}{2}$ (2) $\frac{\sqrt{3}}{3}$ (3) $\frac{\sqrt{6}}{3}$ (4) $\frac{\sqrt{6}}{2}$ (5) $2\sqrt{3}$ (6) $3\sqrt{2}$ (7) $2\sqrt{7}$ (8) $7\sqrt{2}$ (9) $\frac{14\sqrt{3}}{3}$

(10) $5\sqrt{3}$ (12) $\frac{15\sqrt{2}}{2}$ (14) $\frac{\sqrt{10}}{2}$
(11) $\frac{5\sqrt{6}}{2}$ (13) $\frac{\sqrt{10}}{5}$ (15) $\frac{\sqrt{30}}{2}$

Q60. (1) 6 (4) $-4\sqrt{3}$ (7) $6\sqrt{2}$ (10) $-11\sqrt{3}$
(2) $4\sqrt{2}$ (5) 28 (8) 0 (11) $-4\sqrt{2}$
(3) 15 (6) $12\sqrt{2}$ (9) $-6\sqrt{5}$ (12) $10\sqrt{3}$

Q61. (1) $\sqrt{2}+1$ (3) $4\sqrt{3}-6$ (5) -1 (7) $-3\sqrt{2}-\sqrt{14}$
(2) $\sqrt{3}-1$ (4) $-7-4\sqrt{3}$ (6) $\frac{9}{2}\sqrt{2}-3\sqrt{3}$ (8) $\frac{2\sqrt{10}-\sqrt{15}}{5}$

1.8 Expansions

Q62. (1) x^2-2x+1 (6) $x^2-10x+25$ (11) $x^2-\frac{4}{3}x+\frac{4}{9}$ (16) $4x^2+20x+25$
(2) x^2+4x+4 (7) $x^2-3x+2.25$ (12) $4x^2-4x+1$ (17) $\frac{1}{4}x^2-x+1$
(3) x^2-6x+9 (8) $x^2+5x+6.25$ (13) $9x^2+6x+1$ (18) $\frac{9}{4}x^2+6x+4$
(4) $x^2+8x+16$ (9) $x^2+x+\frac{1}{4}$ (14) $4x^2-12x+9$ (19) $25x^2-20x+4$
(5) $x^2-8x+16$ (10) $x^2+\frac{4}{3}x+\frac{4}{9}$ (15) $9x^2+12x+4$ (20) $36x^2+12x+1$

Q63. (1) $a^4-2a^2b+a^2b^2$ (5) $4p^2q^4-12p^4q^3+9p^6q^2$
(2) $x^6+4x^4y^2+4x^2y^4$ (6) $\frac{s^2t^4}{4}-2s^3t^3+4s^4t^2$
(3) $4x^2s^2-12xs^4+9s^6$ (7) $\frac{9}{4}a^2c^2+2a^2c^6+\frac{4}{9}a^2c^{10}$
(4) $9a^4b^6+12a^3b^7+4a^2b^8$ (8) $\frac{4}{9}a-a^4c^3+\frac{9}{16}a^6c^6$

Q64. (1) $6+4\sqrt{2}$ (4) $30-12\sqrt{6}$ (7) $99+60\sqrt{2}$ (10) $114+36\sqrt{10}$
(2) $11-6\sqrt{2}$ (5) $122-56\sqrt{3}$ (8) $201-126\sqrt{2}$ (11) $55-22\sqrt{6}$
(3) $43-24\sqrt{3}$ (6) $182+96\sqrt{3}$ (9) $304-60\sqrt{15}$ (12) $\frac{49}{2}$

Chapter 2
Sets

2.1 Sets and subsets

Q1. (1) For every real number there exists an integer smaller than the real number.
(2) All natural odd powers of -1 are equal -1.
(3) All natural even powers of -1 are equal 1.
(4) There exists such natural number n that each real number s is greater or equal to the sum of n and s.
(5) For every positive real number there is exactly one real number whose square is equal to the number considered.
(6) If 2 divides a natural number then 4 divides it, too.
(7) There is a natural number such that if it is divisible by 2 then it is divisible by 4, too.
(8) There exists a natural number that is not divisible by 2 but it is divisible by 4.
(9) For every integer its power of 2 is an integer, too.
(10) A number is rational whenever its power of 2 is rational.

Q2. (1) $\forall n \in \mathbb{N}\ n \in \mathbb{Z}$ or $n \in \mathbb{N}\ \Rightarrow n \in \mathbb{Z}$, TRUE
(2) $\forall n \in \mathbb{Z}\ n \in \mathbb{Q}$ or $n \in \mathbb{Z}\ \Rightarrow n \in \mathbb{Q}$, TRUE
(3) $\exists x \in \mathbb{R}\ \neg(n > 0) \wedge \neg(n < 0)$, TRUE
(4) $\exists n \in \mathbb{N}\ \neg(n > 0)$, TRUE
(5) $\forall n \in \mathbb{Z}\ (4 \mid n) \Rightarrow (2 \mid n)$, TRUE
(6) $\forall n \in \mathbb{Z}\ ((2 \mid n) \wedge (3 \mid n)) \Rightarrow (6 \mid n)$, TRUE
(7) $\forall x \in \mathbb{R}^+\ \exists y \in \mathbb{R}\ x = y^2$, TRUE
(8) $\forall x \in \mathbb{Z}^+\ \exists y \in \mathbb{Z}\ x = y^2$, FALSE
(9) $\forall x, y \in \mathbb{R}\ \exists z \in \mathbb{Z}^+\ z < |x - y|$, FALSE
(10) $\forall x \in \mathbb{R}\ (x < x^2) \Rightarrow (x < 0)$, FALSE

(11) $\forall x, n \in \mathbb{Z}\ x^{2n} > 0$, FALSE
(12) $\exists x, n \in \mathbb{Z}\ x^{2n+1} \leq 0$, TRUE
(13) $\forall x, y \in \mathbb{R}\ (x < y) \Rightarrow (x^2 < y^2)$, FALSE
(14) $\exists x \in \mathbb{R}^+\ x < x^2$,TRUE
(15) $\forall x \in \mathbb{R}^+\ x > x^2$,FALSE
(16) $\forall n \in \mathbb{Z}\ (2 \mid n) \Rightarrow (4 \mid n^2)$, TRUE
(17) $\forall n \in \mathbb{Z}\ (4 \mid n) \Rightarrow (16 \mid n^2)$, TRUE

Q3. (1) $\{-5, -4, 4, 5\}$
(2) $\{-1, 0, 1\}$
(3) $\{-4, -3, -2, 2, 3, 4\}$
(4) $\{-4, -3, 3, 4\}$
(5) $\{0, 1, 2, 3, 4\}$
(6) $\{2, 3, 4, 5\}$
(7) $\{-1, 0, 1, 3\}$
(8) $\{-1, 0, 1, 2, 3, 4\}$
(9) $\{-3, -1, 1\}$
(10) $\{-2, -\frac{1}{3}, 1\}$
(11) $\emptyset$
(12) $\{-8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2\}$

Q4. (1) $\{0, 1, 2, 3, 4\}$
(2) $\{-8, -7, -6, -5, -4, -3, -2, -1\}$
(3) $\{-2, -1, 0, 1, 2\}$
(4) $\emptyset$
(5) $\mathbb{Z}$
(6) $\mathbb{N}$
(7) $\emptyset$
(8) $\{-5, -4, -3, -2, -1\}$
(9) $\{0, 1, 2, 3\}$
(10) $\{-5, -4, -3, 3, 4, 5\}$
(11) $\{-6, 0, 6\}$
(12) $\{-8, -4, 0, 4, 8\}$

Q5. $\emptyset, \{1\}, \{3\}, \{1, 3\}$

Q6. $\emptyset, \{2\}, \{5\}, \{8\}, \{2, 5\}, \{2, 8\}, \{5, 8\}, \{2, 5, 8\}$

Q7. $\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$

Q8. $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 3\}, \{3, 4\}$

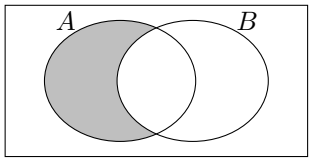
Q9. $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 5\}$

Q10. $\{1, 2, 3, 4\}, \{1, 2, 3, 5\}, \{1, 2, 4, 5\}, \{1, 3, 4, 5\}, \{2, 3, 4, 5\}$

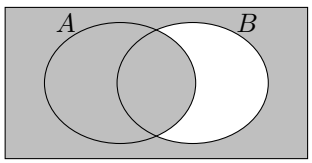
Q11. $\{0\}, \{0, 2\}, \{0, 4\}, \{0, 8\}, \{0, 2, 4\}, \{0, 2, 8\}, \{0, 4, 8\}$

Q12. 15 **Q13.** 16 **Q14.** 8

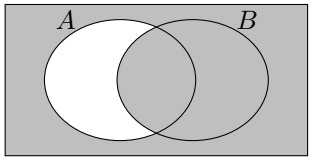
2.2 Venn diagrams

Q15. (1) 

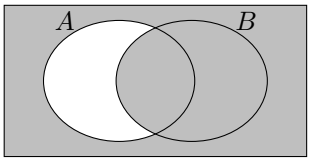
$A \cap B'$

(2) 

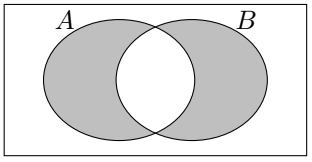
$A \cup B'$

(3) 

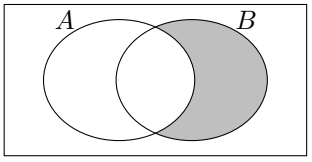
$(A \cap B')'$

(4) 

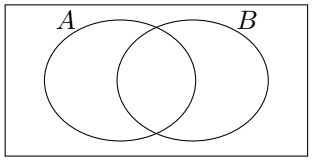
$(A \setminus B)'$

(5) 

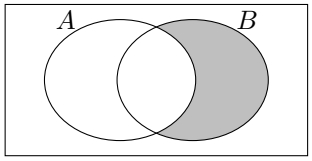
$(A \cup B) \setminus (A \cap B)$

(6) 

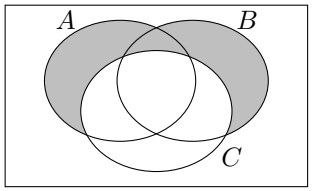
$(A \cup B) \setminus A$

(7) 

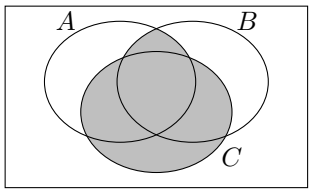
$(A \cap B) \setminus (A \cup B)$

(8) 

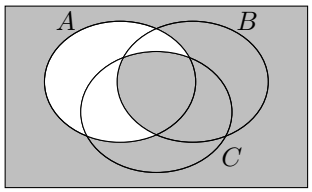
$B \setminus (A \cap B)$

(9) 

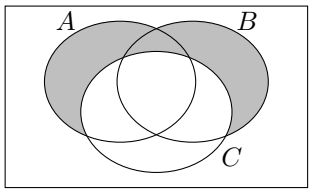
$(A \cup B) \cap C'$

(10) 

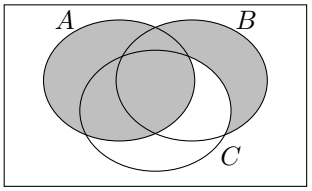
$(A \cap B) \cup C$

(11) 

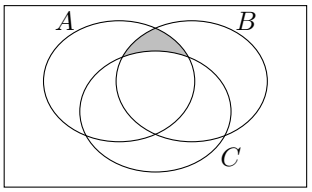
$A' \cup (B \cap C)$

(12) 

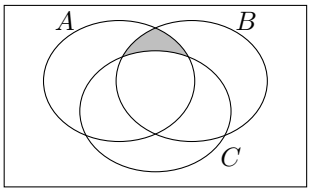
$(A \cup B) \setminus C$

(13) 

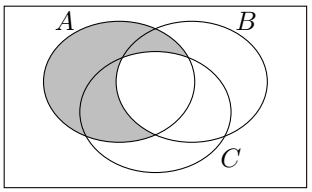
$A \cup (B \setminus C)$

(14) 

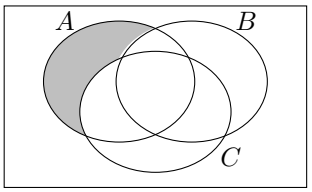
$(A \cap B) \setminus C$

(15) 

$A \cap (B \setminus C)$

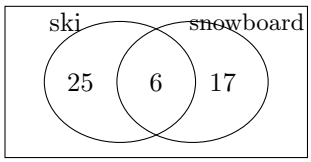
(16) 

$A \setminus (B \cap C)$

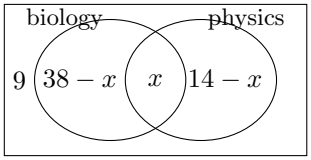
(17) 

$A \setminus (B \cup C)$

Q16. (1) true (2) false (3) false (4) false (5) false (6) true (7) true (8) true (9) true (10) true

Q17. (1) 

(2) 25 (3) 17

Q18. (1) 

$9 + 38 - x + x + 14 - x = 50$
(2) 11 (3) 27

Q19. (1) 6 (2) 2

Q20. 1

Q21. (1) 3 (2) 8

Q22. (1) 28 (2) 3

Q23. (1) 12 (2) 14

Q24. (1) 160 (2) 12

Q25. (1) 20 (2) 4 (3) 11

Q26. (1) 30 (2) 0 (3) 10 (4) 2

Q27. (1) 12 (2) 1

Q28. Biology - 5, Chemistry - 1, English - 2

Q29. 40 **Q30.** 4 **Q31.** 74

2.3 Operations on sets

Q32. (1) $A = \{1, 3, 5, 7, 9, 11\}$
(2) $B = \{0, 2, 4, 6, 8, 10\}$
(3) $\{1, 5, 7, 11\}$
(4) $\{3, 9\}$
(5) $\emptyset$
(6) $\{3, 9\}$
(7) $\{0, 6\}$
(8) $\{0, 2, 4, 6, 8, 10, 12\}$
(9) $\{1, 3, 5, 7, 9, 11, 12\}$
(10) $\{0, 1, 2, \ldots, 10, 11\}$
(11) $\{3, 9\}$
(12) $\{12\}$
(13) $\{12\}$
(14) $\{12\}$
(15) $\{12\}$
(16) 6
(17) 6
(18) 4
(19) 0
(20) 8
(21) 9

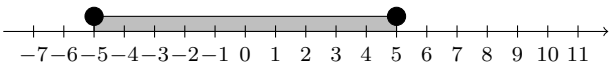
Q33. (1) 11 (2) $\{2, 4, 12\}$ (3) $\{3, 7, 9\}$ (4) $\{0, 6, 8, 10, 14\}$

Q34. (1) 11 (2) $\{3, 6, 18\}$ (3) $\{4, 10, 13\}$ (4) $\{0, 9, 12, 15, 21\}$

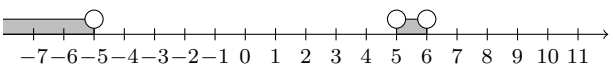
Q35. (1) false
(2) false
(3) true
(4) false
(5) true
(6) false
(7) 11
(8) 7
(9) 3
(10) $\{\pm 15, \pm 9, \pm 6, \pm 3\}$
(11) $\{-8, -4, 4, 8\}$
(12) $\emptyset$
(13) $\emptyset$
(14) $\{\pm 14, \pm 13, \pm 11, \pm 10, \pm 7, \pm 5, \pm 2, \pm 1\}$
(15) B
(16) C
(17) A
(18) C

Q36. (1) 2
(2) 10
(3) 8
(4) $\{6, 8, a, b\}$, where $a, b \in U \setminus A$
(5) $\{1, 6, 10, a, b, c\}$, $a, b, c \in B$
(6) $\{2, 4, 8, a, b, c, d\}$, $a, b, c, d \in U \setminus C$
(7) $\{1, 12\}$
(8) $\{12\}$
(9) $\{1, 12\}$

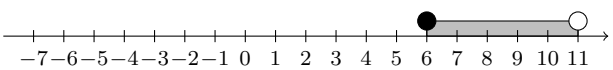
Q37. (1) $[-5, 5]$



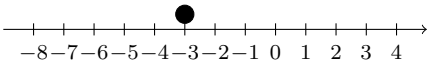
(2) $]-\infty, -5[\cup]5, 6]$



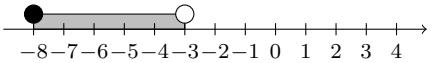
(3) $[6, 11[$



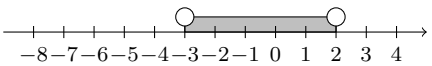
Q38. (1) $\{-3\}$



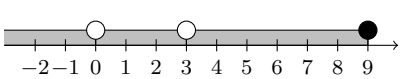
(2) $[-8, -3[$



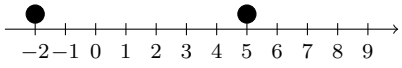
(3) $]-3, 2[$



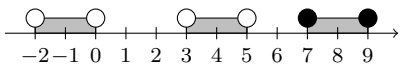
Q39. (1) $]-\infty, 0[\cup]0, 3[\cup]3, 9]$



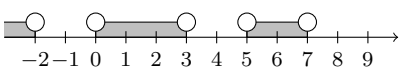
(2) $\{-2, 5\}$



(3) $]-2, 0[\cup]3, 5[\cup[7, 9]$

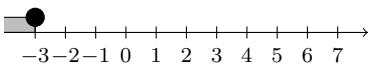


(4) $]-\infty, -2[\cup]0, 3[\cup]5, 7[$

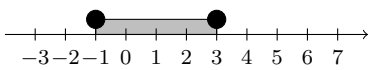


Q40. (1) $\emptyset$

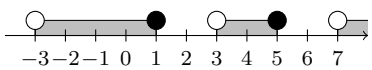
(2) $]-\infty, -3]$



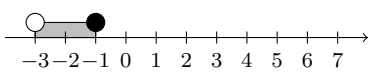
(3) $[-1, 3]$



(4) $]-3, 1]\cup]3, 5]\cup]7, +\infty[$



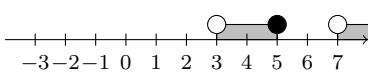
(5) $]-3, -1]$



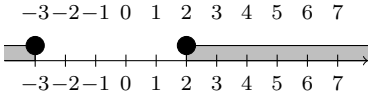
(6) $]-1, +\infty[$



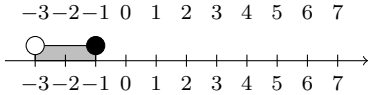
(7) $]3, 5]\cup]7, +\infty[$



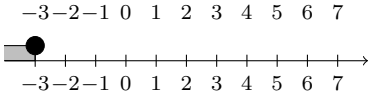
(8) $]-\infty, -3]\cup[2, +\infty[$



(9) $]-3, -1]$



(10) $]-\infty, -3]$



Q41. (1) $(A \cup B)' = A' \cap B'$

(2) $(A \cap B)' = A' \cup B'$

(3) $(A' \cap B)' = A \cup B'$

(4) $(A \cup B')' = A' \cap B$

(5) $A' \cup B' = (A \cap B)'$

(6) $A' \cap B' = (A \cup B)'$

(7) $A' \cup B = (A \cap B')'$

(8) $A \cap B' = (A' \cup B)'$

2.4 Chapter review

Q1. (1) $]-\infty, -4]\cup\{-3\}$

(2) $\{-1, 0, 1, 2\}$

(3) $\{3\}$

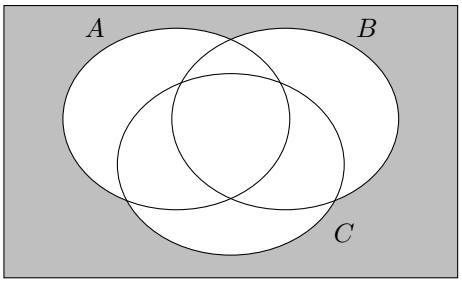
(4) $\{0, 1, 2, 3\}$

(5) $D =]-\infty, -5[\cup[3, +\infty[$

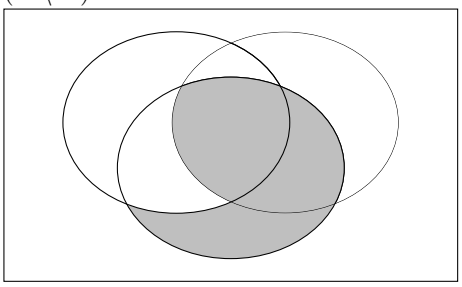
(6) $[-5, 3[$

(7) $]-\infty, -4]\cup[-3, 3[\cup]3, +\infty[$

(8) $[-5, 3[$



$(A \setminus B)' \cap C$

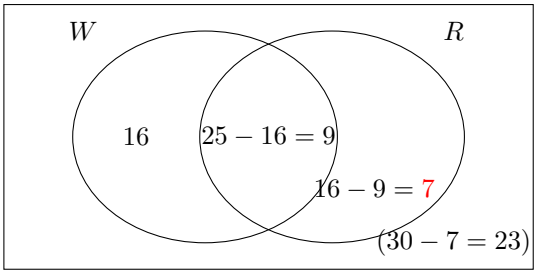


Q2. $\{a\}, \{\circ\}, \{\triangle\}, \{a, \circ\}, \{a, \triangle\}, \{\circ, \triangle\}$, $\{a, \circ, \triangle\}$.
E has 32 subsets.

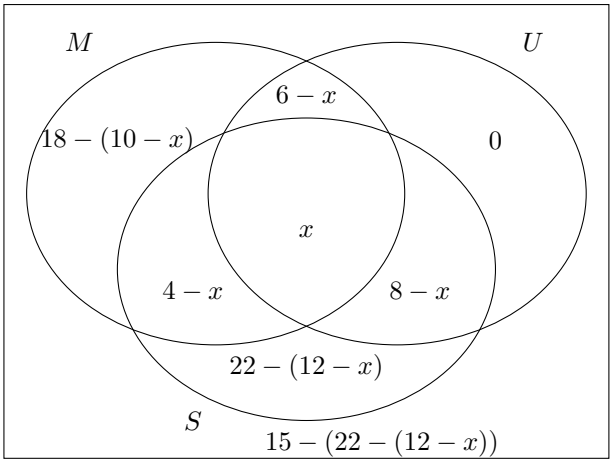
Q3. $A' \cap (B \cup C)'$

Q4. e.g. $(A\setminus(B\cup C))\cup(C\setminus(A\cup B))$ or $(A\cup C)\setminus(A\cap C)\setminus B$

Q5.



Q6.



From total no of students equal 38 we obtain $x = 3$ and hence:

(a) 13 (b) 3 (c) 2

Chapter 3
Lines and vectors

3.1 Basic concepts

Q1. (1) $(6, 7)$
(2) $(8, -1)$
(3) $(3, -5)$
(4) $(-1, 2)$
(5) $(7, -1.5)$
(6) $(-8, -3)$
(7) $(0.5, 6.5)$
(8) $(34, -7)$

Q2. (1) 5
(2) 13
(3) 10
(4) $4\sqrt{2}$
(5) $3\sqrt{10}$
(6) $5\sqrt{2}$
(7) $5\sqrt{5}$
(8) $4\sqrt{5}$
(9) $10\sqrt{2}$
(10) 17

3.2 *Slope-intercept* equation of a line

Q3. (1) $\frac{4}{3}$
(2) $-\frac{3}{4}$
(3) 1
(4) $\frac{1}{3}$
(5) $-\frac{1}{3}$
(6) 2
(7) -2
(8) $-\frac{3}{2}$

Q4. (1) e.g.$(3, 2), (6, 4), (9, 6)$
(2) e.g.$(2, -3), (4, -6), (6, -9)$
(3) e.g.$(2, 0), (3, 2), (4, 4)$
(4) e.g.$(-3, -2), (-2, -5), (-1, -8)$
(5) e.g.$(6, 0), (10, 1), (14, 2)$
(6) e.g.$(-1, -3), (1, -4), (3, -5)$
(7) e.g.$(2, 0), (6, 3), (10, 6)$
(8) e.g.$(2, 0), (5, -5), (8, -10)$
(9) e.g.$(-5, -5), (-2, 2), (1, 9)$
(10) e.g.$(-3, 3), (2, 1), (7, -1)$

Q5. Use a GDC to check your answers.

Q6. (1) yes
(2) yes
(3) no
(4) yes
(5) yes
(6) yes
(7) yes
(8) no
(9) yes
(10) yes

Q7. (1) 7
(2) 6.5
(3) 3
(4) 0
(5) 5.5
(6) -3
(7) 14
(8) -6
(9) 6
(10) -4.5

Q8. (1) $y = 2x - 3$
(2) $y = -3x - 1$
(3) $y = \frac{1}{4}x + 2$
(4) $y = -\frac{1}{2}x - 4$
(5) $y = \frac{3}{4}x - 6$
(6) $y = -\frac{5}{3}x + 5$
(7) $y = \frac{7}{3}x + 3$
(8) $y = -\frac{2}{5}x + 2$
(9) $y = \frac{2}{3}x + 1$
(10) $y = -\frac{3}{2}x + 3$

Q9. (1) $y = 2x - 2$
(2) $y = -3x + 3$
(3) $y = \frac{1}{4}x - \frac{1}{2}$
(4) $y = -\frac{1}{2}x + \frac{5}{2}$
(5) $y = \frac{3}{4}x - \frac{1}{2}$
(6) $y = -\frac{5}{3}x + \frac{1}{3}$
(7) $y = \frac{7}{3}x + 1$
(8) $y = -\frac{2}{5}x - \frac{8}{5}$
(9) $y = \frac{2}{3}x + 3$
(10) $y = -\frac{3}{2}x - 4$

Q10. (1) $y = -\frac{1}{2}x + 3$ (2) $y = \frac{1}{3}x - 7$ (3) $y = -4x - 14$ (4) $y = 2x + 6$ (5) $y = -\frac{4}{3}x + 12$ (6) $y = \frac{3}{5}x + 5\frac{2}{5}$ (7) $y = -\frac{3}{7}x - 1\frac{6}{7}$ (8) $y = \frac{5}{2}x - 20\frac{1}{2}$ (9) $y = -\frac{3}{2}x + 12$ (10) $y = \frac{2}{3}x + 3\frac{2}{3}$

Q11. (1) $m > 0$ (2) $m > -1$ (3) $m > 2$ (4) $m > 2.5$

Q12. (1) $m > 4$ (2) $m < -\frac{3}{4}$ (3) no such m (4) $-2 < m < 2$

Q13. (i) $91.4\frac{km}{h}$
(ii) between 1.5 and 2hrs: $96\frac{km}{h}$
(iii) between 1.5 and 2.5hrs: $95\frac{km}{h}$

3.3 *General* equation of a line

Q14. (1) $y = -\frac{2}{3}x - 2\frac{1}{2}$ (2) $y = -2x + 1\frac{1}{3}$ (3) $y = \frac{1}{2}x + 1\frac{1}{2}$ (4) $y = -\frac{3}{4}x - \frac{1}{3}$ (5) $y = 3x - 2$ (6) $y = \frac{5}{3}x - 1\frac{1}{3}$ (7) $y = \frac{2}{5}x - 3\frac{1}{2}$ (8) $y = -\frac{7}{3}x - \frac{1}{2}$ (9) $y = -\frac{1}{4}x + \frac{1}{2}$ (10) $y = \frac{3}{2}x + 1\frac{1}{3}$

Q15. (1) $6x - 3y - 4 = 0$ (2) $3x + y - 2 = 0$ (3) $x - 4y + 2 = 0$ (4) $x + 2y + 3 = 0$ (5) $9x - 12y + 4 = 0$ (6) $5x + 3y - 4 = 0$ (7) $14x - 6y - 3 = 0$ (8) $4x + 10y - 35 = 0$ (9) $4x - 6y - 15 = 0$ (10) $9x + 6y + 8 = 0$

Q16. (1) $4x + 6y + 15 = 0$ (2) $6x + 2y - 3 = 0$ (3) $x - 2y + 3 = 0$ (4) $5x + 6y + 4 = 0$ (5) $3x - y - 2 = 0$ (6) $5x - 3y - 4 = 0$ (7) $4x - 5y - 35 = 0$ (8) $14x + 6y + 3 = 0$ (9) $x + 4y - 2 = 0$ (10) $9x - 8y + 6 = 0$

Q17. (1) $4x + 6y - 20 = 0$ or $2x + 3y - 10 = 0$
(2) $6x + 3y = 0$ or $2x + y = 0$
(3) $x - 2y - 1 = 0$
(4) $9x + 12y + 27 = 0$ or $3x + 4y + 9 = 0$
(5) $3x - y - 14 = 0$
(6) $5x - 3y + 29 = 0$
(7) $4x - 10y - 2 = 0$ or $2x - 5y - 1 = 0$
(8) $14x + 6y - 138 = 0$ or $7x + 3y - 69 = 0$
(9) $x + 4y + 2 = 0$
(10) $9x - 6y + 42 = 0$ or $3x - 2y + 14 = 0$

Q18. (1) $6x - 4y - 4 = 0$ or $3x - 2y - 2 = 0$
(2) $3x - 6y - 45 = 0$ or $x - 2y - 15 = 0$
(3) $2x + y + 8 = 0$
(4) $12x - 9y + 36 = 0$ or $4x - 3y + 12 = 0$
(5) $x + 3y - 18 = 0$
(6) $3x + 5y - 3 = 0$
(7) $10x + 4y + 24 = 0$ or $5x + 2y + 12 = 0$
(8) $6x - 14y - 26 = 0$ or $3x - 7y - 13 = 0$
(9) $4x - y - 43 = 0$
(10) $6x + 9y + 15 = 0$ or $2x + 3y + 5 = 0$

Q19. (1) 10 (2) $3\sqrt{13}$ (3) $2\sqrt{5}$ (4) 19.5 (5) $\sqrt{10}$ (6) $\frac{9}{2}\sqrt{45}$ (7) $2\sqrt{116}$ (8) $2\sqrt{13}$ (9) $2\sqrt{17}$ (10) $3\sqrt{29}$ (11) $3\sqrt{17}$ (12) $\frac{1}{2}\sqrt{401}$ (13) $2\sqrt{101}$ (14) $2\sqrt{37}$ (15) $\sqrt{13}$

3.4 Vectors

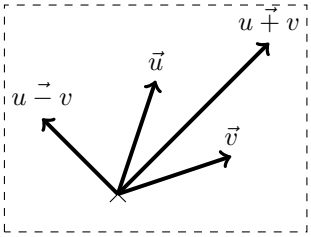
Q20. (1) $\begin{pmatrix}-4\\4\end{pmatrix}$ (2) $\begin{pmatrix}2\\4\end{pmatrix}$ (3) $\begin{pmatrix}3\\6\end{pmatrix}$ (4) $\begin{pmatrix}1\\-3\end{pmatrix}$ (5) $\begin{pmatrix}2\\6\end{pmatrix}$ (6) $\begin{pmatrix}-3\\7\end{pmatrix}$ (7) $\begin{pmatrix}-5\\-10\end{pmatrix}$ (8) $\begin{pmatrix}8\\-10\end{pmatrix}$ (9) $\begin{pmatrix}8\\3\end{pmatrix}$ (10) $\begin{pmatrix}-4\\8\end{pmatrix}$

Q21. (1) $(-18,-1)$ (2) $(3,-7)$ (3) $(1,0)$ (4) $(6,10)$ (5) $(1,3)$ (6) $(-1,-9)$ (7) $(-6,2)$ (8) $(0,-1)$ (9) $(-5,10)$ (10) $(15,4)$

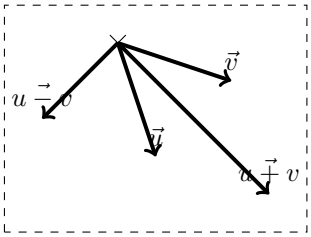
Q22. (1) $\begin{pmatrix}-5\\4\end{pmatrix}$ (2) $\begin{pmatrix}-2\\-6\end{pmatrix}$ (3) $\begin{pmatrix}-1\\-1\end{pmatrix}$ (4) $\begin{pmatrix}1\\-13\end{pmatrix}$ (5) $\begin{pmatrix}1\\-10\end{pmatrix}$ (6) $\begin{pmatrix}-12\\-9\end{pmatrix}$

Q23. (1) $\begin{pmatrix}-8\\11\end{pmatrix}$ (2) $\begin{pmatrix}5\\-7\end{pmatrix}$ (3) $\begin{pmatrix}-3\\3\end{pmatrix}$ (4) $\begin{pmatrix}-1\\1\end{pmatrix}$ (5) $\begin{pmatrix}-33\\-2\end{pmatrix}$ (6) $\begin{pmatrix}-7\\-13\end{pmatrix}$ (7) $\begin{pmatrix}13\\5\end{pmatrix}$ (8) $\begin{pmatrix}15\\-20\end{pmatrix}$ (9) $\begin{pmatrix}2\\2\end{pmatrix}$ (10) $\begin{pmatrix}-23\\-37\end{pmatrix}$

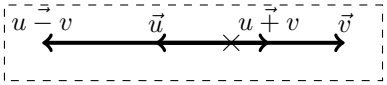
Q24. (1)



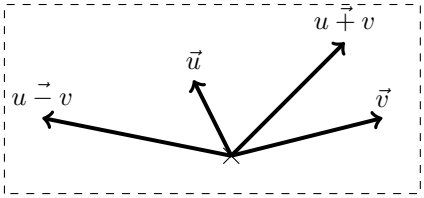
(2)



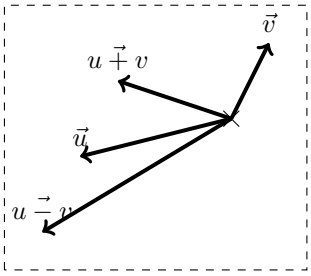
(3)



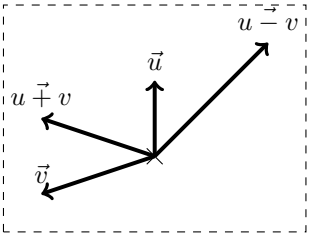
(4)



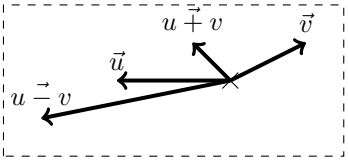
(5)



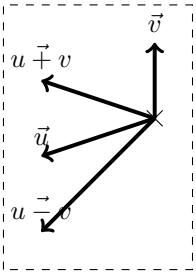
(6)



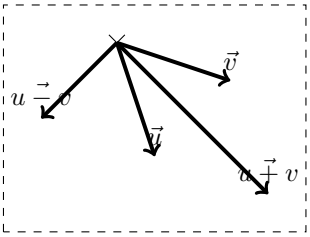
(7)



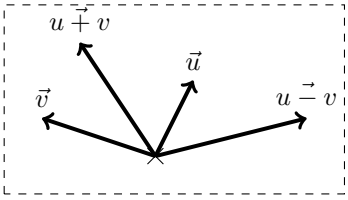
(8)



(9)



(10)



Q25. (1) 10 left, 4 up
(2) 19 right, 18 up
(3) 4 left, 4 up
(4) 7 right, 22 down
(5) 5 left, 6 up
(6) 18 right, 11 down
(7) 6 left, 6 down
(8) 8 left, 7 up
(9) 7 left, 9 up
(10) 1 right, 9 down

Q26. (1) $\pm\begin{pmatrix}6\\-8\end{pmatrix}$ (2) $\pm\begin{pmatrix}12\\9\end{pmatrix}$ (3) $\pm\begin{pmatrix}-48\\20\end{pmatrix}$ (4) $\pm\begin{pmatrix}6\sqrt{5}\\12\sqrt{5}\end{pmatrix}$ (5) $\pm\begin{pmatrix}\frac{18}{\sqrt{5}}\\\frac{24}{\sqrt{5}}\end{pmatrix}$ (6) $\pm\begin{pmatrix}6\sqrt{10}\\2\sqrt{10}\end{pmatrix}$ (7) $\pm\begin{pmatrix}-4\sqrt{13}\\6\sqrt{13}\end{pmatrix}$ (8) $\pm\begin{pmatrix}2\sqrt{5}\\-6\sqrt{5}\end{pmatrix}$ (9) $\pm\begin{pmatrix}-3\sqrt{2}\\3\sqrt{2}\end{pmatrix}$ (10) $\pm\begin{pmatrix}3\\-3\sqrt{3}\end{pmatrix}$

Q27. (1) 3 (2) 2 (3) 5 (4) -1 (5) 3 (6) $-\frac{10}{3}$

Q28. (1) $\pm\begin{pmatrix}12\\9\end{pmatrix}$ (3) $\pm\begin{pmatrix}15\\36\end{pmatrix}$ (5) $\pm\begin{pmatrix}\frac{12}{\sqrt{5}}\\-\frac{9}{\sqrt{5}}\end{pmatrix}$ (7) $\pm\begin{pmatrix}9\sqrt{13}\\6\sqrt{13}\end{pmatrix}$ (9) $\pm\begin{pmatrix}\frac{9}{2}\sqrt{2}\\\frac{9}{2}\sqrt{2}\end{pmatrix}$

(2) $\pm\begin{pmatrix}6\\-8\end{pmatrix}$ (4) $\pm\begin{pmatrix}8\sqrt{5}\\-4\sqrt{5}\end{pmatrix}$ (6) $\pm\begin{pmatrix}3\sqrt{10}\\-9\sqrt{10}\end{pmatrix}$ (8) $\pm\begin{pmatrix}12\sqrt{5}\\4\sqrt{5}\end{pmatrix}$ (10) $\pm\begin{pmatrix}4\sqrt{3}\\4\end{pmatrix}$

Q29. (1) e.g. $\begin{pmatrix}0\\-1\end{pmatrix}, \begin{pmatrix}3\\1\end{pmatrix}, \begin{pmatrix}6\\3\end{pmatrix}$

(2) e.g. $\begin{pmatrix}0\\-3\end{pmatrix}, \begin{pmatrix}2\\-6\end{pmatrix}, \begin{pmatrix}4\\-9\end{pmatrix}$

(3) e.g. $\begin{pmatrix}0\\-3\end{pmatrix}, \begin{pmatrix}1\\-1\end{pmatrix}, \begin{pmatrix}2\\1\end{pmatrix}$

(4) e.g. $\begin{pmatrix}0\\6\end{pmatrix}, \begin{pmatrix}1\\3\end{pmatrix}, \begin{pmatrix}2\\0\end{pmatrix}$

(5) e.g. $\begin{pmatrix}0\\-9\end{pmatrix}, \begin{pmatrix}4\\-8\end{pmatrix}, \begin{pmatrix}8\\-7\end{pmatrix}$

(6) e.g. $\begin{pmatrix}0\\4\end{pmatrix}, \begin{pmatrix}2\\3\end{pmatrix}, \begin{pmatrix}4\\2\end{pmatrix}$

(7) e.g. $\begin{pmatrix}0\\8\end{pmatrix}, \begin{pmatrix}4\\11\end{pmatrix}, \begin{pmatrix}8\\14\end{pmatrix}$

(8) e.g. $\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}3\\-4\end{pmatrix}, \begin{pmatrix}6\\-9\end{pmatrix}$

(9) e.g. $\begin{pmatrix}0\\3\end{pmatrix}, \begin{pmatrix}3\\10\end{pmatrix}, \begin{pmatrix}6\\17\end{pmatrix}$

(10) e.g. $\begin{pmatrix}0\\-4\end{pmatrix}, \begin{pmatrix}5\\-6\end{pmatrix}, \begin{pmatrix}10\\-8\end{pmatrix}$

(11) e.g. $\begin{pmatrix}0\\3\end{pmatrix}, \begin{pmatrix}3\\-1\end{pmatrix}, \begin{pmatrix}6\\-5\end{pmatrix}$

(12) e.g. $\begin{pmatrix}0\\2\end{pmatrix}, \begin{pmatrix}2\\-3\end{pmatrix}, \begin{pmatrix}4\\-8\end{pmatrix}$

(13) e.g. $\begin{pmatrix}0\\-5\end{pmatrix}, \begin{pmatrix}-2\\-6\end{pmatrix}, \begin{pmatrix}-4\\-7\end{pmatrix}$

(14) e.g. $\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}12\\-6\end{pmatrix}, \begin{pmatrix}24\\-13\end{pmatrix}$

(15) e.g. $\begin{pmatrix}0\\-2\end{pmatrix}, \begin{pmatrix}-1\\-5\end{pmatrix}, \begin{pmatrix}-2\\-8\end{pmatrix}$

(16) e.g. $\begin{pmatrix}0\\-4\end{pmatrix}, \begin{pmatrix}-3\\-9\end{pmatrix}, \begin{pmatrix}-6\\-14\end{pmatrix}$

(17) e.g. $\begin{pmatrix}0\\2\end{pmatrix}, \begin{pmatrix}-10\\-2\end{pmatrix}, \begin{pmatrix}-20\\-6\end{pmatrix}$

(18) e.g. $\begin{pmatrix}0\\6\end{pmatrix}, \begin{pmatrix}5\\3\end{pmatrix}, \begin{pmatrix}10\\0\end{pmatrix}$

(19) e.g. $\begin{pmatrix}0\\-3\end{pmatrix}, \begin{pmatrix}4\\-4\end{pmatrix}, \begin{pmatrix}8\\-5\end{pmatrix}$

(20) e.g. $\begin{pmatrix}0\\-1\end{pmatrix}, \begin{pmatrix}-6\\-10\end{pmatrix}, \begin{pmatrix}-12\\-19\end{pmatrix}$

3.5 Simultaneous equations

Q30. (1) $x=\frac{10}{7}, y=\frac{12}{7}$ (2) $x=1, y=2$ (3) $x=2, y=-3$ (4) $x=-2, y=1$ (5) $x=-\frac{3}{2}, y=2$ (6) $x=-6, y=9$ (7) $x=3, y=2$ (8) $x=-3, y=2$ (9) $x=-1, y=3$ (10) $x=-1, y=3$ (11) $x=0, y=2$ (12) $x=-1, y=0$ (13) $x=-4, y=-3$ (14) $x=-\frac{12}{13}, y=\frac{34}{13}$ (15) $x=-\frac{9}{2}, y=1$ (16) $x=8, y=-7$

Q31. (1) $x=-1, y=1$ (2) $x=-0.5, y=1.5$ (3) $x=2, y=1$ (4) $x=1, y=1$ (5) $x=-\frac{11}{5}, y=\frac{18}{5}$ (6) $x=-2, y=4$ (7) no solutions (8) $x=-\frac{7}{2}, y=-\frac{4}{3}$

3.6 Applications of linear equations and vectors

Q32. (i) $A(-4,-3), B(4,1)$ (ii) $4\sqrt{5}$ (iii) 30

Q33. (i) $A(-4,2), B(8,-2)$ (ii) $4\sqrt{10}$ (iii) 24

Q34. (i) $-6, 4.5$ (ii) $(3,3)$ (iii) 15.75

Q35. (i) $12\sqrt{5}$ (ii) 30

Q36. (i) $12\sqrt{10}$ (ii) 60

Q37. 22.5

Q38. 19.5

Q39. 12 girls and 8 boys

Q40. 38 cars, 14 motorcycles

Q41. 18

Q42. 34

Q43. 6 buses, 24 cars

Q44. $y = 0.358x - 1.59$

Q45. $\left(\frac{5}{3}, -2\right)$

Q46. (i) $y = 2.8x + 8$
(ii) 41.60 pln
(iii) 7.85 km

Q47. (i) 42.6 mln
(ii) 28.4
(iii) 48.3
(iv) 2203

Q48. (i) c - 7.21mph
(ii) 12 miles East, 10 miles North of O, YES - at 2pm
(iii) 15 miles East, 12 miles North of O, NO - c will arrive first

Q49. (i) 11 km 200 m
(ii) 17 minutes

Q50. (i) 347 km
(ii) 3h 48mins
(iii) $91.2\frac{km}{h}$

Q51. 20

Q52. $(6, 5)$ and $(-2, 9)$

Q53. $(2, 4)$ or $(8, -8)$

Q54. $(-4, 0)$

Q55. $C = (9, 1)$,
$D = (-1, 7)$

Q56. $(2, 3)$

Q57. $A = (3, 1)$,
$B = (2, 4)$,
$C = (-1, 3)$

Q58. $C = (-5, 3)$,
$D = (-8, 1)$

Q59. (i) $x - 2y - 9 = 0$
(ii) $2\sqrt{5}$
(iii) $C = (3, -3)$,
$D = (-3, -6)$

Q60. (1) $\vec{AB} = \begin{pmatrix} 9 \\ -6 \end{pmatrix}$
(2) $|\vec{AB}| = 3\sqrt{13}$.
(3) $BC = 2\sqrt{13}$
(4) $\vec{BC} = \pm \begin{pmatrix} 4 \\ 6 \end{pmatrix}$
(5) either
$(6, 4)$
$(-3, 10)$
or
$(-2, -8)$
$(-11, -2)$

Q61. (1) $\vec{AB} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}$
(2) $|\vec{AB}| = 4\sqrt{10}$
(3) $\frac{5}{2}\sqrt{10}$
(4) $3\sqrt{10}$
(5) $(3, 8)$, $(-6, 5)$

Q62. $(3, -5)$, $(8, 10)$

Q63. (i) $3x + 2y + 4 = 0$
(ii) $2x - 3y + 7 = 0$
(iii) $(1, 3)$
(iv) $(3, 0)$
(v) $2\sqrt{13}$
(vi) $(2, -5)$

Q64. (i) $(-5, -1)$
(ii) $(0, -3.5)$, $(-4, 4.5)$

Q65. (i) hint: the product of the slopes is -1
(ii) $(x - 0.5)^2 + (y - 4)^2 = 21.25$
(iii) $(-4, 5)$

Q66. (i) $x + 2y + 3 = 0$
(ii) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $2\sqrt{5}$
(iii) $3\sqrt{5}$
(iv) $(-5, 6)$

Q67. (i) $(-3, -1.5)$
(ii) 58.5

Q68. (i) either $(9, 7)$ and $(3, 11)$ or
$(-5, -1)$ and $(1, -5)$
(ii) either $(-7, -4)$ and $(-1, -8)$ or
$(11, 10)$ and $(5, 14)$

Q69. $(-3, 3)$

3.7 Chapter review

non-calculator questions

Q1. (i) $(2.5, -4.5)$
(ii) $(0, -12)$
(iii) 75
(iv) 37.5

Q2. (i) $2x + y + 3 = 0$
(ii) $(-2, 1)$
(iii) $A : -12, B : 0.5$
(iv) $C : 4, D : -3.5$
(v) ABD;
they have the same base (BD),
but the heights: $AP > CP$
(either $AP = 5\sqrt{5} > 3\sqrt{5} = CP$ or
e.g. $|\vec{AP}| = |\begin{pmatrix} 10 \\ 5 \end{pmatrix}| > 10$
and $|\vec{PS}| = |\begin{pmatrix} 6 \\ 3 \end{pmatrix}| < 9$
so $AP > 10 > 9 > CP$)
(vi) $(-8.5, 4)$

Q3. (i) $12\sqrt{5} + 3\sqrt{10}$
(ii) $(10.5, -1)$
(iii) $(\frac{20}{3}, -\frac{5}{3})$
(iv) $y = 2x - 15$
(v) $8\sqrt{5} + 2\sqrt{10}$

Q4. (i) $3\sqrt{13}$
(ii) $(-5, 1)$
(iii) $(-8, -3.5)$ and $(-2, 5.5)$

calculator questions

Q5. (i) $A(-6, -2.5)$, $B(4, 2.5)$
(ii) $(1, 5)$
(iii) $x - 2y + 9 = 0$ or $y = \frac{1}{2}x + \frac{9}{2}$
(iv) $(-3.54, 2.73)$
(v) 25.9
(vi) 3.58
(vii) 29.1

Q6. (i) $\pm\begin{pmatrix}6.84\\13.7\end{pmatrix}$
(ii) $\pm\begin{pmatrix}1.75\\8.77\end{pmatrix}$
(iii) $\begin{pmatrix}0\\0\end{pmatrix}$
(iv) $\begin{pmatrix}-4\\-8\end{pmatrix}$

Q7. (i) $A(12.2, 0)$, $B(0.935)$
(ii) $(6.12, 4.68)$
(iii) 114

Chapter 4
Functions

4.1 Basic properties

Q1. (1) yes (2) yes (3) yes (4) no (5) no (6) yes

Q2. (1) (i) $-4 < x \leq 4$
(ii) $-3 \leq y \leq 3$
(iii) $-2, 2.5$
(iv) $]-5, 1]$
(v) $[1, 4]$
(vi) —
(vii) $0, 1.5$

(2) (i) $-5 \leq x < 5$
(ii) $-3 \leq y < 2$
(iii) 3
(iv) $[-5, -1]$, $[2, 5[$
(v) —
(vi) $[-1, 2]$
(vii) $[-1, 2]$

(3) (i) $-5 < x \leq 5$
(ii) $1 \leq y < 2$
(iii) —
(iv) —
(v) $]-5, -3]$, $]-3, -1]$, $]-1, 1]$, $]1, 3]$, $]3, 5]$
(vi) —
(vii) —

(4) (i) $-5 \leq x < 5$
(ii) $\{-2, -1, 0, 1, 2, \}$
(iii) $[-1, 1[$
(iv) —
(v) —
(vi) $[-5, -3[$, $[-3, -1[$, $[-1, 1[$, $[1, 3[$, $[3, 5[$
(vii) —

(5) (i) $x \geq -5$
(ii) $-2 \leq y \leq 2$
(iii) -4
(iv) $[-5, -3]$
(v) $[-3, -1]$
(vi) $x \geq -1$
(vii) $-3.5 < x < -1$

(6) (i) $-4 < x \leq 2$
(ii) $-3 < y \leq 3$
(iii) -1
(iv) $-4 < x \leq 2$
(v) —
(vi) —
(vii) $-4 < x \leq -2$

(7) (i) $-5 \leq x \leq 2$
(ii) $-2 \leq y \leq 3$
(iii) $-3.5, -1$
(iv) $[-5, -2[$, $[-2, 2]$
(v) —
(vi) —
(vii) 1

(8) (i) $x \geq -5$
(ii) $y \geq -1$
(iii) $-3, -1.5$
(iv) $[-2, -1]$, $[4, +\infty[$
(v) $[-5, -2]$
(vi) $[-1, 4]$
(vii) $[-3, -1.5]$

(9) (i) $[-4, 5[$
(ii) $[-1, 2[$
(iii) $-\frac{10}{3}, -\frac{1}{2}, 4$
(iv) $[-4, -2[$, $[-2, 1[$
(v) $[3, 5[$
(vi) $[1, 3[$
(vii) $-\frac{8}{3}, 3$

(10) (i) $x \leq 5$
(ii) $y \leq 2$
(iii) $-3.5, -1.5$
(iv) $x \leq -3$
(v) $[-2, -1[$, $[-1, 0]$
(vi) $[-3, -2] \cup [0, 5]$
(vii) $x \leq -4$

(11) (i) $x \geq -4$
(ii) $y \leq 4$
(iii) 3
(iv) $[-2, -1]$
(v) $[-4, -2]$, $[-1, +\infty[$
(vi) –
(vii) $-4, -1.5, 0$

(12) (i) $-4 < x \leq 5$
(ii) $-2 \leq y < 3$
(iii) $-2.5, -1, 2$
(iv) $[-2, 1]$, $[4, 5]$
(v) $]-4, -2]$, $[1, 4]$
(vi) —
(vii) $-2, 3, 5$

(13) (i) $x \leq 4$
(ii) $y \leq 3$

(iii) $-4, -2$
(iv) $]-\infty, -3]$, $[-2, 0]$, $[3, 4]$
(v) $[-3, -2]$, $[0, 3]$
(vi) —
(vii) $-3, -1, 1, 3.5$

(14) (i) $-4 \leq x < 5$
(ii) $-2 < y \leq 2$
(iii) $-2, -1, 4$
(iv) $[2, 3]$
(v) $[-4, -3[$, $[-3, -2]$, $]-2, -1]$, $]-1, 0[$, $[0, 2]$, $[3, 5[$
(vi) —
(vii) $-4, -3, 0, 3$

(15) (i) $-5 < x < 3$
(ii) $-2 \leq y \leq 2$
(iii) $-4, -1, 2$
(iv) $]-5, -3]$, $[1, 3[$
(v) $[-3.1]$
(vi) —
(vii) -3

(16) (i) $-4 < x < 4$
(ii) $-3 < y \leq 3$
(iii) $-3, 1$
(iv) $]-4, -1]$, $[2, 3]$
(v) $[-1, 2]$, $[3, 4[$
(vi) —
(vii) —

(17) (i) $-4 \leq x \leq 3$
(ii) $-2 \leq y \leq 3$
(iii) $-3.5, 1$
(iv) $[-4, -2]$
(v) $[-2, 3]$
(vi) —
(vii) $[-4, 3[\cup]0, 3]$

(18) (i) $-4 < x \leq 3$
(ii) $-2 \leq y \leq 2$
(iii) $-2, 0, 3$
(iv) $]-4, -1]$, $[1, 3]$
(v) $[-1, 1]$
(vi) —
(vii) $]-4, -3[\cup]-0.5, 2[$

(19) (i) $-5 < x \leq 3$
(ii) $1 \leq y \leq 3$
(iii) —
(iv) $]-5, -3]$, $[-1, 1]$
(v) $[-3, -1]$, $[1, 3]$
(vi) —
(vii) $]-5, -3[\cup]-3, 0[\cup]2, 3]$

(20) (i) $-5 \leq x \leq 3$
(ii) $-2 \leq y \leq 2$
(iii) 2, between -4 and -3, between -2 and -1
(iv) $[-3, 0]$, $[2, 3]$
(v) $[-5, -3]$, $[0, 2]$
(vi) —
(vii) $]-5, -1[\cup]1, 3[$

(21) (i) $-4 < x \leq 3$
(ii) $0 \leq y \leq 2$
(iii) 3
(iv) $]-4, -3]$
(v) $[-3, -2]$, $[1, 3]$
(vi) $[-2, 1[$
(vii) $[-2, 1[\cup\{2\}$

(22) (i) $x \leq 2$
(ii) $y \geq -1$
(iii) $-3, -1$
(iv) $[-2, 0]$
(v) $]-\infty, -2]$
(vi) $[0, 2]$
(vii) $]-3, -1[$

(23) (i) $-4 < x \leq 4$
(ii) $-1 < y \leq 1$
(iii) $-3, -1, 1, 3$
(iv) $]-4, -2]$, $]-2, 0]$, $]0, 2]$, $]2, 4]$
(v) —
(vi) —
(vii) $\{-2, 0, 2, 4\}$

(24) (i) $-3 \leq x < 1$
(ii) $\{-1, 0, 1, 2\}$
(iii) $[-1, 0[$
(iv) —
(v) —
(vi) $[-3, -2[$, $[-2, -1[$, $[-1, 0[$, $[0, 1[$
(vii) $[-1, 1[$

(25) (i) $-4 \leq x \leq 4$
(ii) $-3 \leq y \leq 2$
(iii) $-1, 0, 3$
(iv) $[-4, -1]$, $[0, 1[$
(v) $]-1, 0]$, $[1, 4]$
(vi) —
(vii) $[-4, 1[\cup[1, 2]$

(26) (i) $-4 < x \leq 3$
(ii) $-2 \leq y \leq 2$
(iii) $-3.5, 2$
(iv) $]-4, 1]$, $]-1, 1]$
(v) $[1, 3]$
(vi) —
(vii) $-4 < x \leq 3$

(27) (i) $-3 \leq x < 2$, $2 < x \leq 3$
(ii) $y = -1$, $0 \leq y \leq 2$
(iii) -2
(iv) $[-2, -1]$
(v) $[1, 2[$
(vi) $[-3, -2[$, $[-1, 1]$, $]2, 3]$
(vii) $[-3, -2]\cup]2, 3]$

(28) (i) $-4 < x \leq -1$, $0 < x \leq 2$
(ii) $-3 < y \leq 2$
(iii) $-2, 2$
(iv) $]-4, -1]$, $]0, 1]$
(v) $[1, 2]$
(vi) —
(vii) $]-2, -1]\cup]0, 2[$

(29) (i) $x > -4$
(ii) $y \geq -1$
(iii) 0
(iv) $]-3, -1]$, $[1, +\infty[$
(v) $]-4, -3]$, $]0, 1]$
(vi) —
(vii) $\{-1\}\cup]0, +\infty[$

(30) (i) $-2 < x \leq 3$
(ii) $-2 \leq y \leq 1$
(iii) 2
(iv) $[1, 3]$
(v) $]-2, -1]$
(vi) $[-1, 0]$, $]0, 1]$
(vii) $[-1, 0]$

Q3. (1) domain: $\mathbb{R}$, range: $[0, +\infty[$
(2) domain: $\mathbb{R}$, range: $\mathbb{R}$
(3) domain: $\mathbb{R} \setminus \{0\}$, range: $\mathbb{R} \setminus \{0\}$
(4) domain: $[0, +\infty[$, range: $[0, +\infty[$
(5) domain: $\mathbb{R}$, range: $\mathbb{R}$
(6) domain: $\mathbb{R}$, range: $[0, +\infty[$

Q4. (1) $-\frac{5}{2}$ (2) 3 (3) $\frac{1}{3}$ (4) ± 3 (5) $\frac{1}{2}$ (6) $-3, 1$ (7) $-\frac{3\pi}{2}$ (8) $-\frac{1}{3}$ (9) -5.5 (10) ± 2 (11) $-3, 1$ (12) — (13) $-2\sqrt{3}$ (14) 4.5 (15) 2 (16) 1 (17) $\frac{1}{2}, \frac{7}{2}$ (18) 9

Q5. (1) domain: $\mathbb{R}$, range: $\mathbb{R}$
(2) domain: $]-\infty, 3]$, range: $[0, +\infty[$
(3) domain: $\mathbb{R} \setminus \{1\}$, range: $\mathbb{R} \setminus \{3\}$
(4) domain: $\mathbb{R}$, range: $[-4, +\infty[$
(5) domain: $\mathbb{R}$, range: $[-2, +\infty[$
(6) domain: $\mathbb{R}$, range: $\mathbb{R}$
(7) domain: $[-\frac{1}{3}, +\infty[$, range: $[0, +\infty[$
(8) domain: $\mathbb{R} \setminus \{-4\}$, range: $\mathbb{R} \setminus \{-2\}$
(9) domain: $\mathbb{R}$, range: $[-9, +\infty[$
(10) domain: $\mathbb{R}$, range: $[3, +\infty[$
(11) domain: $\mathbb{R}$, range: $\mathbb{R}$
(12) domain: $[0, +\infty[$, range: $[-3, +\infty[$
(13) domain: $\mathbb{R}$, range: $[0, +\infty[$
(14) domain: $\mathbb{R}$, range: $]-\infty, 3]$
(15) domain: $[0, \frac{1}{4}[\cup]\frac{1}{4}, +\infty[$, range: $]-\infty, 4[\cup [7, +\infty[$

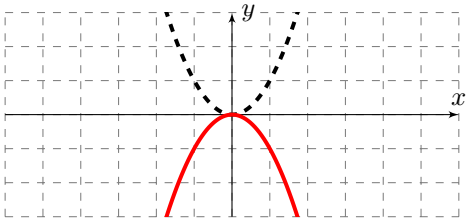
Q6. (1) 1 (2) 3 (3) 3.5 (4) -40 (5) 16 (6) 0 (7) 5π (8) 5 (9) -3 (10) $-\frac{11}{9}$ (11) $\frac{13}{9}$ (12) 4.2 (13) 18 (14) $2\sqrt{6} - 3$ (15) -16 (16) -27 (17) $\frac{2}{3}$ (18) $-\frac{1}{3}$

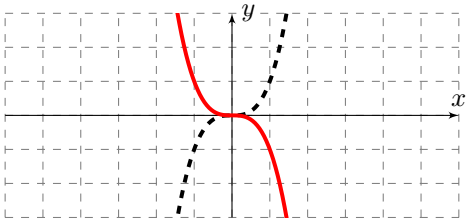
Q7. (1) $f(-x) = -2x + 5$
(2) $f(-x) = \sqrt{3 + x}$
(3) $f(-x) = 3 - \frac{2}{x+1}$
(4) $f(-x) = 9 - x^2$
(5) $f(-x) = (2x + 1)^2$
(6) $f(-x) = |1 - x| - 2$
(7) $f(-x) = -\frac{2}{3}x + \pi$
(8) $f(-x) = \sqrt{1 - 3x}$
(9) $f(-x) = \frac{3}{x-4} - 2$
(10) $f(-x) = x^2 - 4$
(11) $f(-x) = (1 - x)^2 - 4$
(12) $f(-x) = 3 + |4 + 2x|$
(13) $f(-x) = -\sqrt{3}x + 6$
(14) $f(-x) = \sqrt{-2x} - 3$
(15) $f(-x) = -x^3 - 8$
(16) $f(-x) = -(x + 1)^3$
(17) $f(-x) = 3 - |2x + 4|$
(18) $f(-x) = \frac{\sqrt{-x}-3}{2\sqrt{-x}-1}$

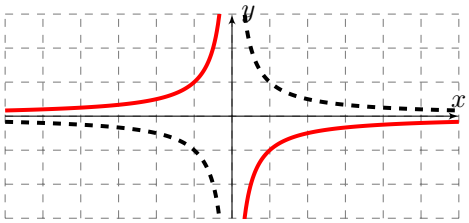
Q8. (1) O (2) N (3) O (4) E (5) E (6) N (7) O (8) O (9) N (10) E (11) N (12) E (13) E (14) O (15) E (16) N (17) E (18) N

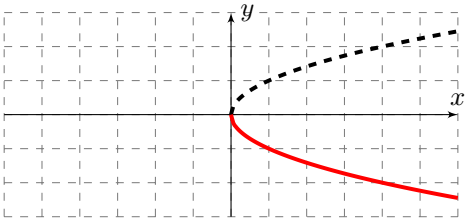
4.2 Transformations of graphs of functions

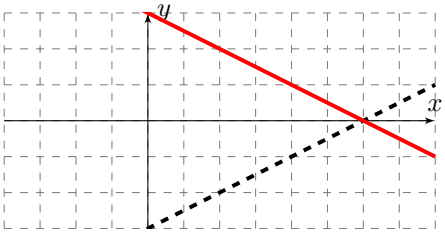
Q9. Graphs of $y = f(x)$ (black, dashed) and $y = -f(x)$ (red, solid)

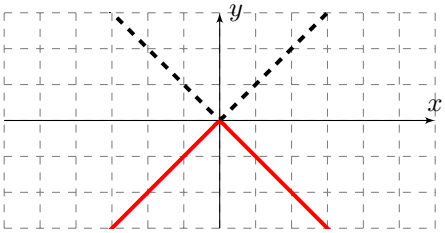
(1)


(2)


(3)


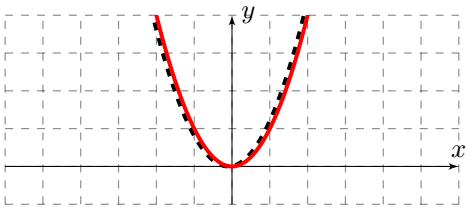
(4)


(5)


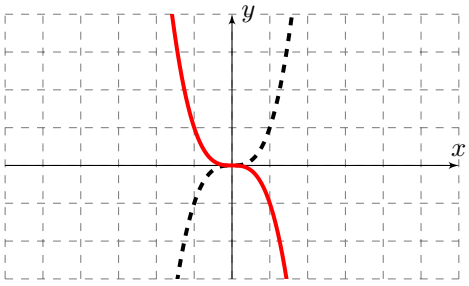
(6)


Q10. Graphs of $y = f(x)$ (black, dashed) and $y = f(-x)$ (red, solid)

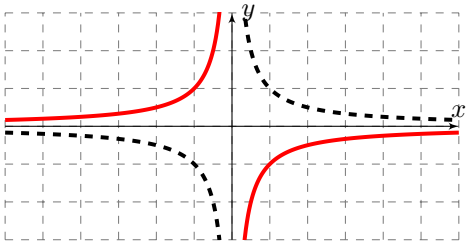
(1)



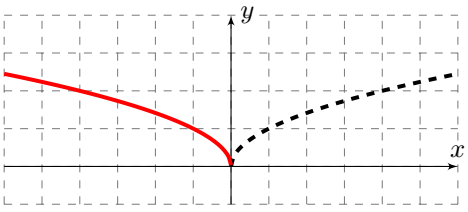
(2)



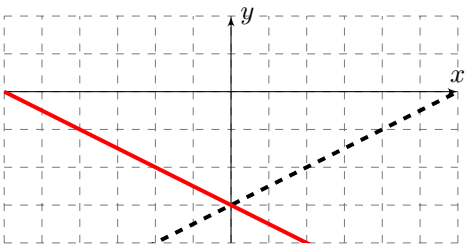
(3)



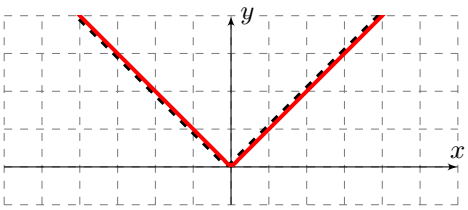
(4)



(5)

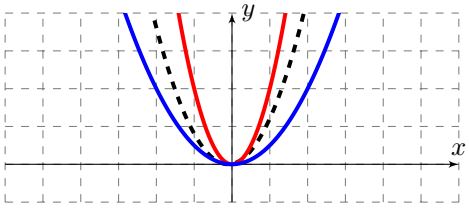


(6)

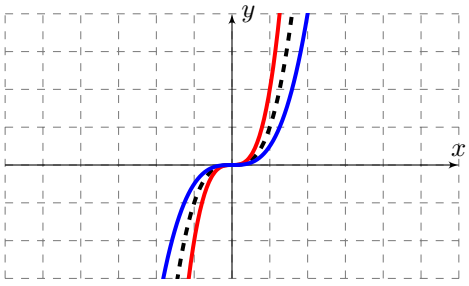


Q11. Graphs of $y = f(x)$ (black, dashed), $y = 2f(x)$ (red, solid) and $y = \frac{1}{2}f(x)$ (blue, solid)

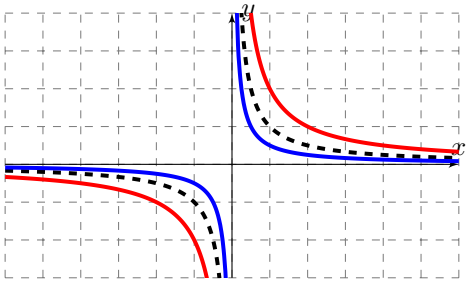
(1)



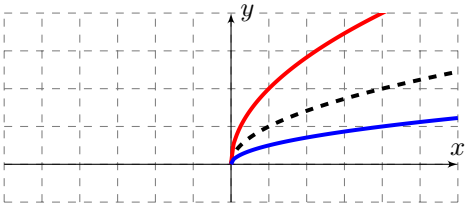
(2)



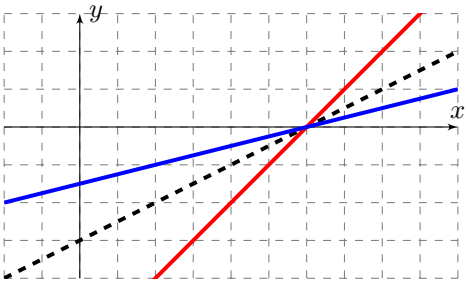
(3)



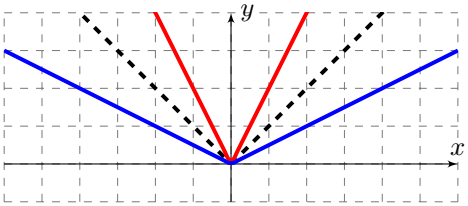
(4)



(5)

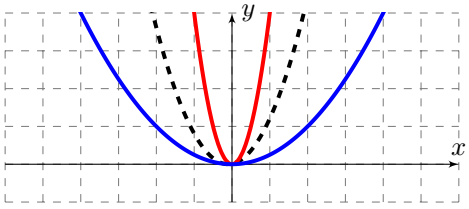


(6)

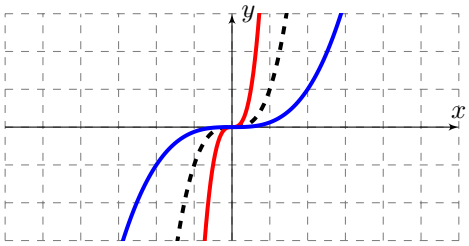


Q12. Graphs of $y = f(x)$ (black, dashed), $y = f(2x)$ (red, solid) and $y = f(\frac{1}{2}x)$ (blue, solid)

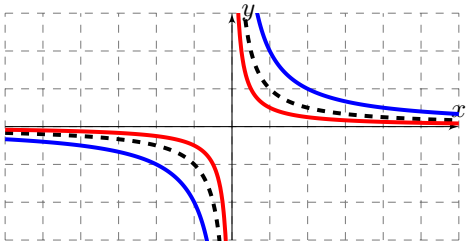
(1)



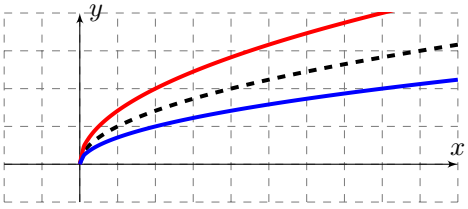
(2)



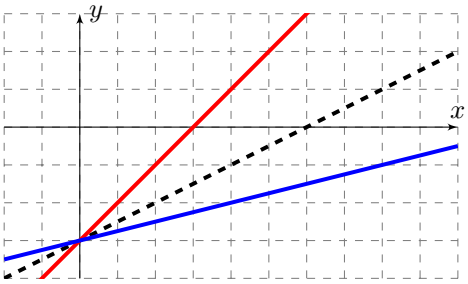
(3)



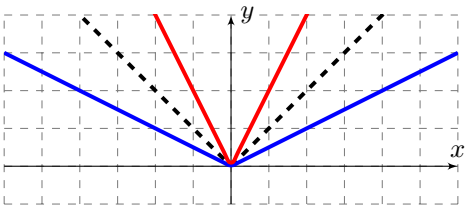
(4)



(5)

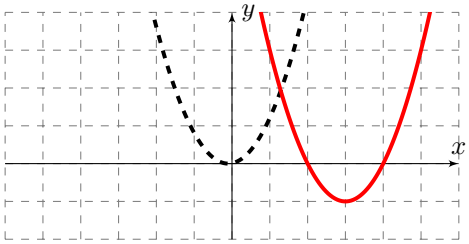


(6)



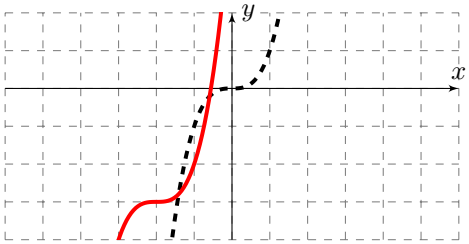
Q13. Graphs of $y = f(x)$ (black, dashed) and $y = g(x)$ (red, solid).

(1)



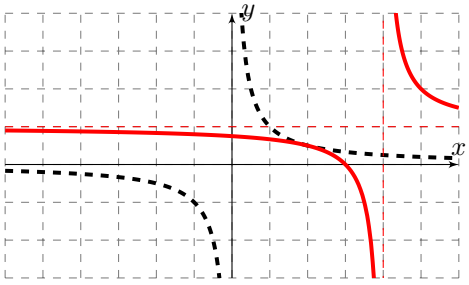
$g(x) = (x-3)^2 - 1$
$x \in \mathbb{R}$, $y \geq -1$

(2)



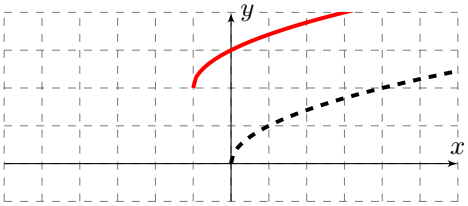
$g(x) = (x+2)^3 - 3$
$x, y \in \mathbb{R}$

(3)



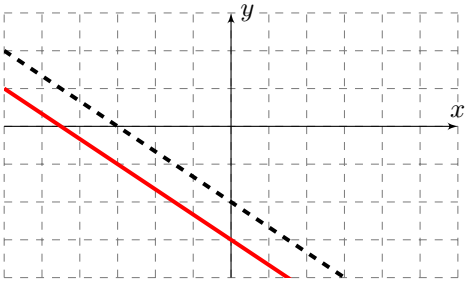
$g(x) = \frac{1}{x-4} + 1$
$x \in \mathbb{R} \setminus \{4\}$, $y \in \mathbb{R} \setminus \{1\}$

(4)



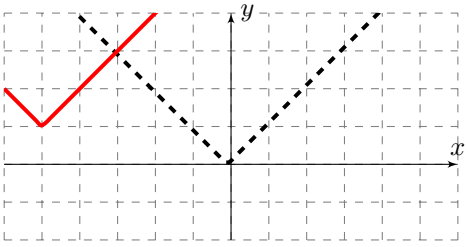
$g(x) = \sqrt{x+1} + 2$
$x \geq -1$, $y \geq 2$

(5)



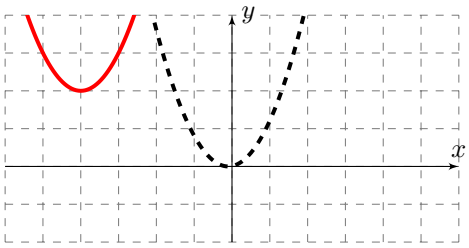
$g(x) = -\frac{2}{3}x - 3$
$x, y \in \mathbb{R}$

(6)



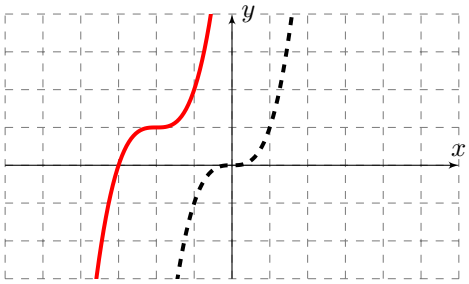
$g(x) = |x+5| + 1$
$x \in \mathbb{R}$, $y \geq 1$

(7)



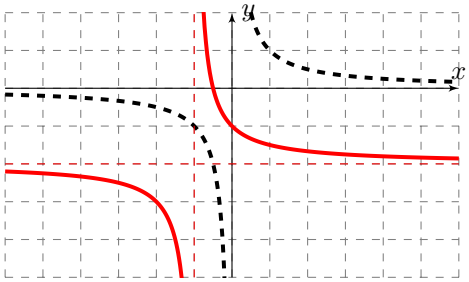
$g(x) = (x+4)^2 + 2$
$x \in \mathbb{R}$, $y \geq 2$

(8)



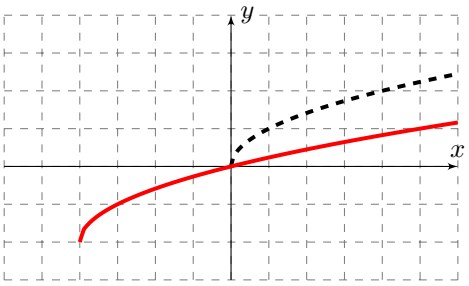
$g(x) = (x+2)^3 + 1$
$x, y \in \mathbb{R}$

(9)



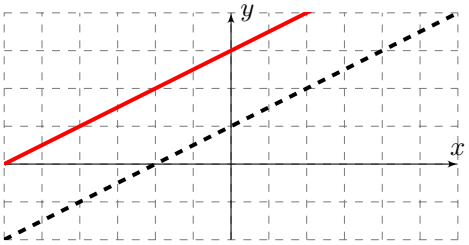
$g(x) = \frac{1}{x+1} - 2$
$x \in \mathbb{R} \setminus \{-1\}$, $x \in \mathbb{R} \setminus \{-2\}$

(10)



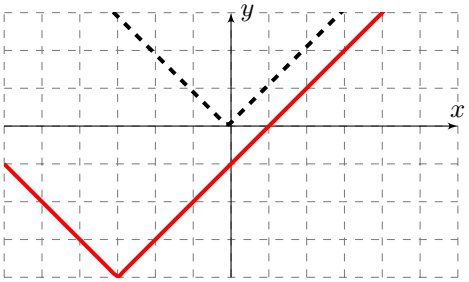
$g(x) = \sqrt{x+4} - 2$
$x \geq -4$, $y \geq -2$

(11)



$g(x) = \frac{1}{2}x + 3$
$x, y \in \mathbb{R}$

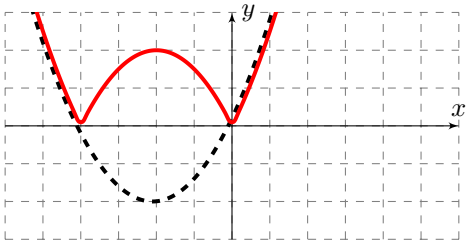
(12)



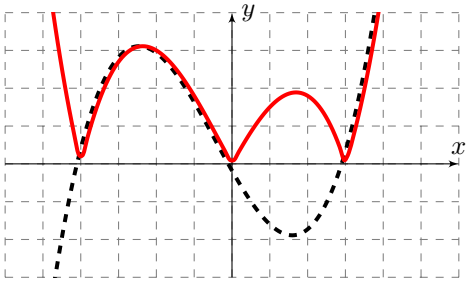
$g(x) = |x+3| - 4$
$x \in \mathbb{R}$, $y \geq -4$

Q14. $y = f(x)$ (black, dashed), $y = |f(x)|$ (red, solid)

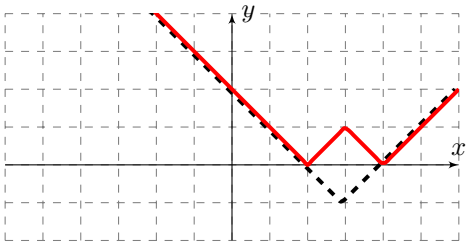
(1)



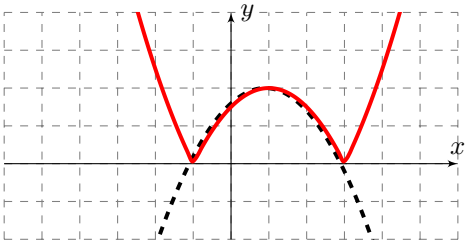
(2)



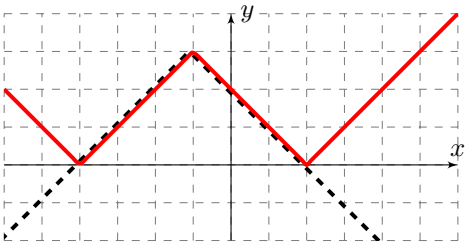
(3)



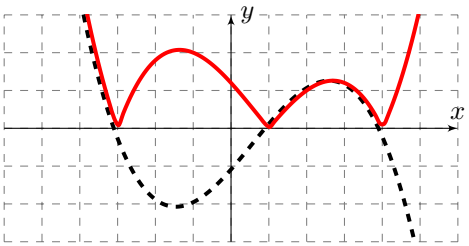
(4)



(5)

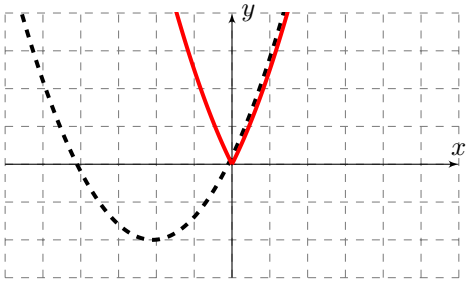


(6)

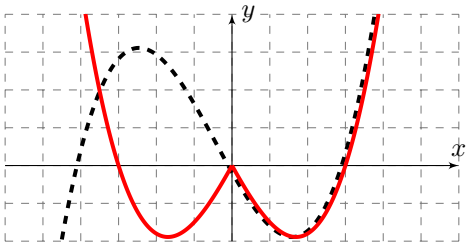


Q15. graph of $y = f(x)$ (dashed, black) and of $y = f(|x|)$ (solid, red)

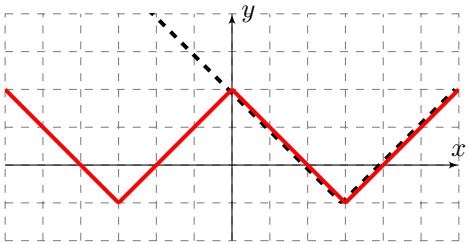
(1)



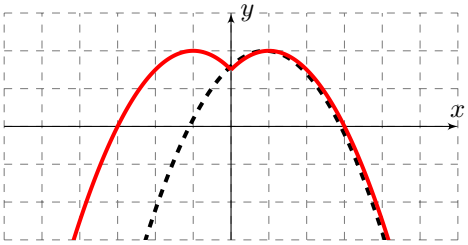
(2)



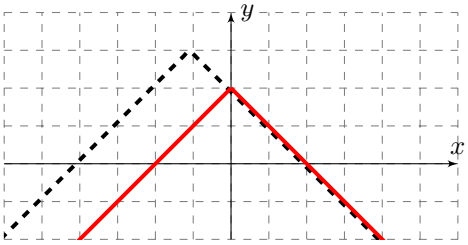
(3)



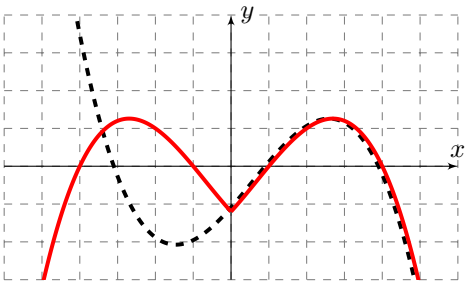
(4)



(5)



(6)



Q16. (1) (i) $y = x^2$

(ii) translation by $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

(iii) $y = (x-3)^2 + 1$

(2) (i) $y = x^2$

(ii) translation by $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

(iii) $y = (x-2)^2 - 4$

(3) (i) $y = x^2$

(ii) translation by $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$

(iii) $y = (x+1)^2 - 4$

(4) (i) $y = x^3$

(ii) translation by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

(iii) $y = (x-4)^3 + 1$

(5) (i) $y = x^3$

(ii) translation by $\begin{pmatrix} -5 \\ 1 \end{pmatrix}$

(iii) $y = (x+5)^3 + 1$

(6) (i) $y = x^3$

(ii) reflection in x-axis or in y-axis

(iii) $y = -x^3$

(7) (i) $y = x^3$

(ii) reflection in x-axis or in y-axis

followed by translation by $\begin{pmatrix} -5 \\ 1 \end{pmatrix}$

(iii) $y = -(x+5)^3 + 1$

(8) (i) $y = \sqrt{x}$

(ii) translation by $\begin{pmatrix} -5 \\ -2 \end{pmatrix}$

(iii) $y = \sqrt{x+5} - 2$

(9) (i) $y = \sqrt{x}$

(ii) reflection in y-axis

(iii) $y = \sqrt{-x}$

(10) (i) $y = \sqrt{x}$

(ii) reflection in y-axis

followed by translation by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

(iii) $y = \sqrt{-x} - 3$

(11) (i) $y = |x|$

(ii) translation by $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

(iii) $y = |x-2| - 3$

(12) (i) $y = |x|$

(ii) reflection in x axis

followed by translation by $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(iii) $y = -|x-2| + 1$

(13) (i) $y = \sqrt{x}$

(ii) translation by $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$

(iii) $y = \sqrt{x+2} - 2$

(14) (i) $y = \sqrt{x}$

(ii) reflection in y-axis

followed by translation by $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$

or translation by $\begin{pmatrix}-2\\-2\end{pmatrix}$
followed by reflection in y-axis

(iii) $y = \sqrt{-x+2} - 2$

(15) (i) $y = \sqrt{x}$
(ii) reflection in x-axis followed by shift 4 left and 1 up
(iii) $y = -\sqrt{x+4} + 1$

(16) (i) $y = \frac{1}{x}$
(ii) reflection in x-axis or in y-axis
(iii) $y = -\frac{1}{x}$

(17) (i) $y = \frac{1}{x}$
(ii) translation 2 right
(iii) $y = \frac{1}{x-2}$

(18) (i) $y = \frac{1}{x}$
(ii) translation by $\begin{pmatrix}-2\\-1\end{pmatrix}$
(iii) $y = \frac{1}{x+2} - 1$

(19) (i) $y = \frac{1}{x}$
(ii) reflection in x-axis or in y-axis
followed by translation by $\begin{pmatrix}-2\\-1\end{pmatrix}$
(iii) $y = -\frac{1}{x+2} - 1$

(20) (i) $y = \frac{1}{x}$
(ii) reflection in x-axis or in y-axis
followed by translation by $\begin{pmatrix}1\\2\end{pmatrix}$
(iii) $y = -\frac{1}{x-1} + 2$

(21) (i) $y = x^2$
(ii) vertical stretch by -2
followed by translation by $\begin{pmatrix}3\\3\end{pmatrix}$
(iii) $y = -2(x-3)^2 + 3$

(22) (i) $y = x^2$
(ii) vertical stretch by $\frac{1}{2}$
followed by translation by $\begin{pmatrix}-1\\-2\end{pmatrix}$
(iii) $y = \frac{1}{2}(x+1)^2 - 2$

(23) (i) $y = |x|$
(ii) vertical stretch by $-\frac{2}{3}$
followed by translation by $\begin{pmatrix}3\\2\end{pmatrix}$
(iii) $y = -\frac{2}{3}|x-3| + 2$

(24) (i) $y = x^3$
(ii) vertical stretch by $-\frac{1}{4}$
followed by translation by $\begin{pmatrix}-3\\2\end{pmatrix}$
(iii) $y = -\frac{1}{4}(x+3)^3 + 2$

(25) (i) $y = \sqrt{x}$
(ii) horizontal dilation by $-\frac{1}{2}$
followed by translation 2 down
(iii) $y = \sqrt{-2x} - 2$

(26) (i) $y = \sqrt{x}$
(ii) vertical stretch by 2
followed by translation by $\begin{pmatrix}-1\\-3\end{pmatrix}$
(iii) $y = 2\sqrt{x+1} - 3$

(27) (i) $y = \sqrt{x}$
(ii) reflection in x-axis followed by horizontal stretch by 2
and translation 3 up
(iii) $y = -\sqrt{\frac{x}{2}} + 3$

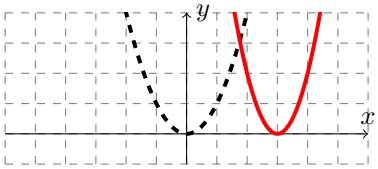
(28) (i) $y = \frac{1}{x}$
(ii) vertical / horizontal dilation by $\frac{1}{2}$
followed by translation 1 down
(iii) $y = \frac{1}{2x} - 1$

(29) (i) $y = \frac{1}{x}$
(ii) vertical / horizontal stretch by -2
followed by shift 2 right 1 up
(iii) $y = \frac{-2}{x-2} + 1$

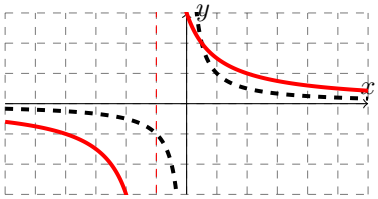
(30) (i) $y = x^2$
(ii) shift 3 right and 4 down followed by reflection of the part below x-axis in the axis
(iii) $y = |(x-3)^2 - 4|$

Q17. Graphs of $y = f(x)$ (black, dashed) and $y = g(x)$ (red, dotted).

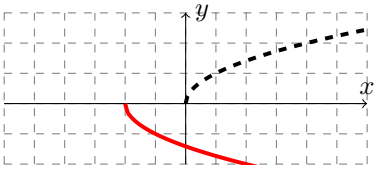
(1) vertical stretch by 2, translation 3 right



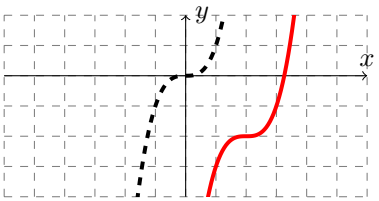
(2) vertical stretch by 3, translation 1 left



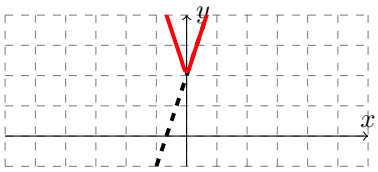
(3) reflection in x-axis, translation 2 left



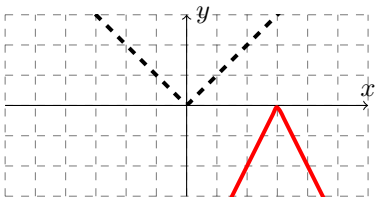
(4) translation 2 right, 2 down



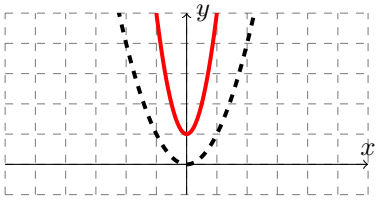
(5) reflection in y axis of the part for $x > 0$



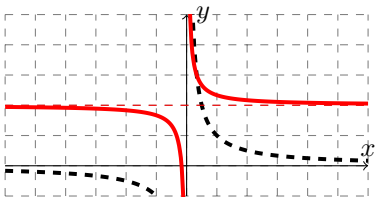
(6) vertical stretch by -2, translation 3 right



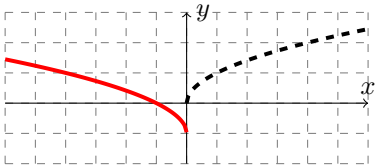
(7) horizontal dilation by $\frac{1}{2}$, translation 1 up



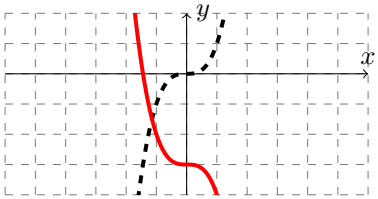
(8) horizontal dilation by $\frac{1}{3}$, translation 2 up



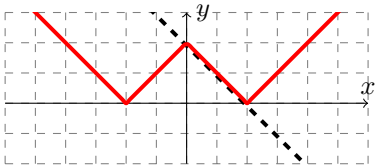
(9) reflection in y-axis, translation 1 down



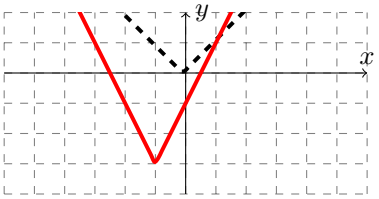
(10) reflection in x/y-axis followed by translation 3 down



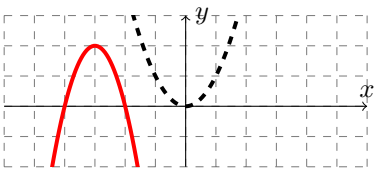
(11) reflection in y-axis of the part $x > 0$ followed by reflection in x-axis of the part $y < 0$



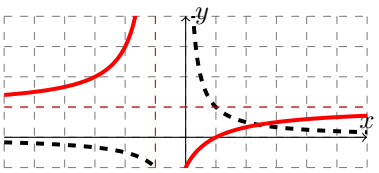
(12) vertical stretch by 2 followed by translation 1 left, 3 down



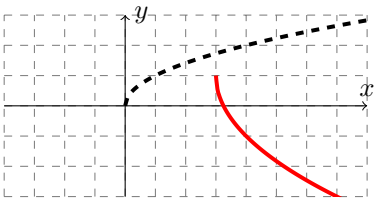
(13) vertical stretch by -2 followed by translation 3 left, 2 up



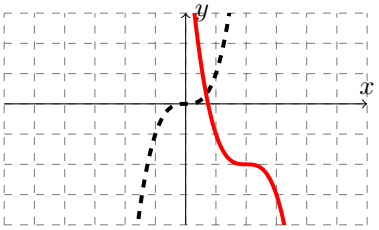
(14) vertical dilation by -2 followed by translation 1 left, 1 up



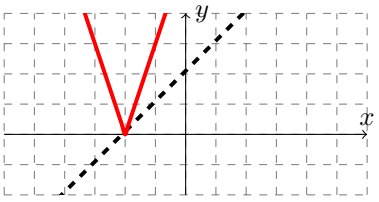
(15) vertical dilation by -2 followed by translation 3 right 1 up



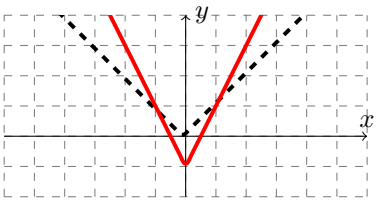
(16) reflection in x-axis followed by translation 2 right 2 down



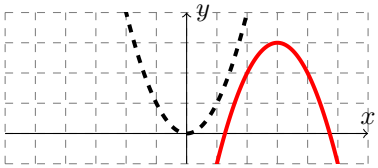
(17) reflection in x-axis of the part $y < 0$ followed by vertical stretch by 2



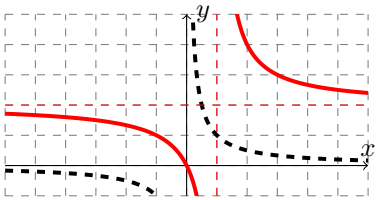
(18) horizontal dilation by $\frac{1}{2}$, translation 1 down



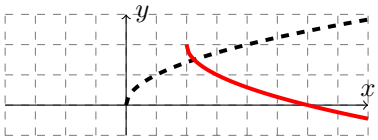
(19) reflection in x-axis followed by translation 3 right, 3 up



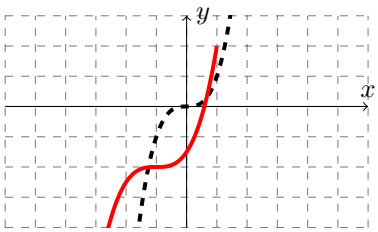
(20) vertical stretch by 2 followed by translation 1 right, 2 up



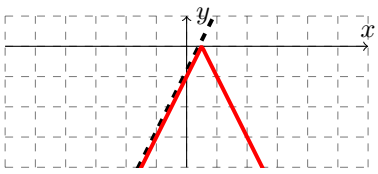
(21) reflection in x-axis followed by translation 2 right 2 up



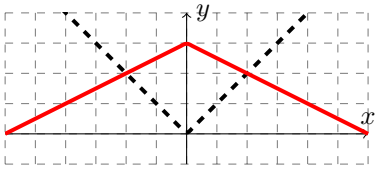
(22) vertical dilation by $\frac{1}{2}$ followed by translation 1 left 2 down



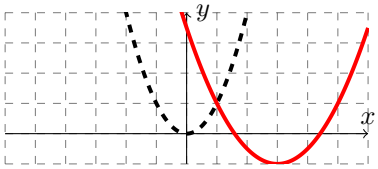
(23) reflection in x-axis of the part $y < 0$ followed by reflection in x-axis



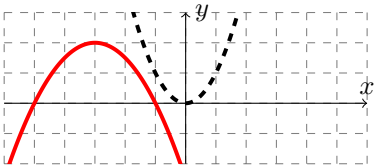
(24) reflection in x-axis and horizontal dilation by 2 followed by translation 3 up



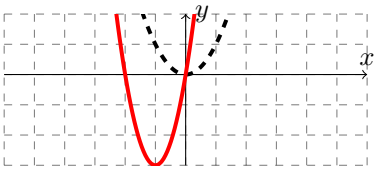
(25) vertical dilation by $\frac{1}{2}$ followed by translation 3 right, 1 down



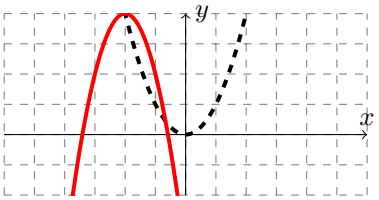
(26) vertical dilation by $-\frac{1}{2}$ followed by translation 3 left, 2 up



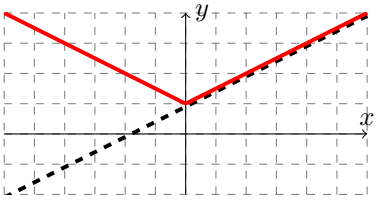
(27) vertical dilation by 3 followed by translation 1 left, 3 down



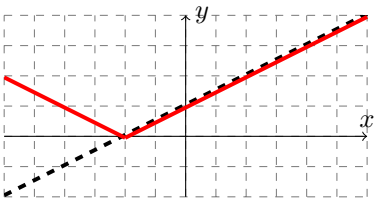
(28) vertical dilation by -2 followed by translation 2 left, 4 up



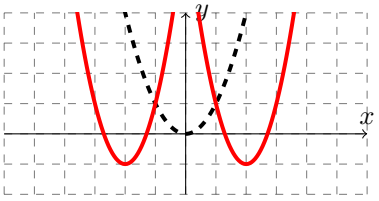
(29) reflection of the part left of y-axis in the axis



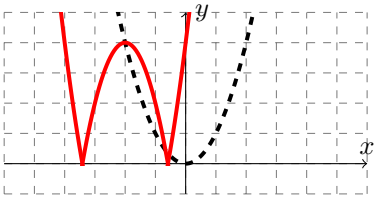
(30) reflection of the part below x-axis in the axis



(31) vertical stretch by 2, then shift 2 right and 1 down and reflection of the part right of y-axis in the axis

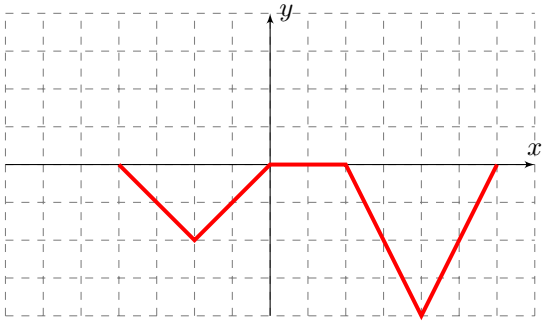


(32) vertical stretch by 2, then shift by $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$ and reflection of the part below x-axis in the axis

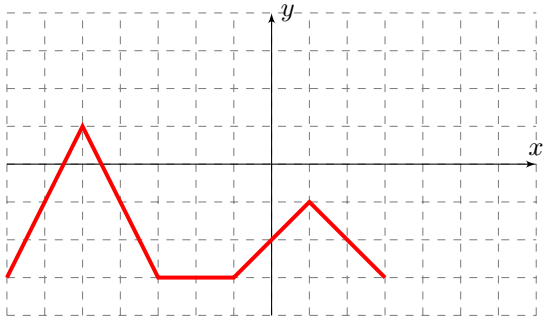


Q18. (1) $A' = (4, 4)$,
(2) $A' = (-3, 2)$,
(3) $A' = (-1.5, 3)$,
(4) $A' = (-3, -4)$,
(5) $A' = (2, 7)$,
(6) $A' = (-8, -5)$.

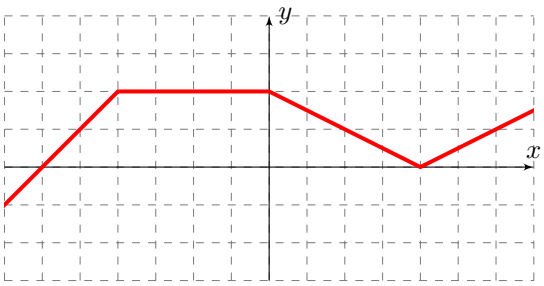
Q19. (i) (1) $y = f(-x) - 2$



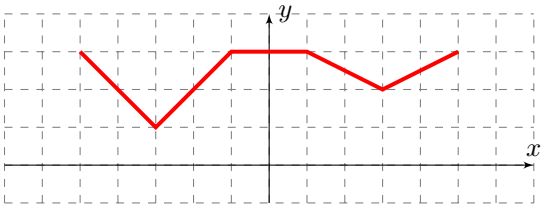
(2) $y = -f(x + 1) - 1$



(3) $y = f(\frac{x}{2})$



(4) $y = \frac{1}{2}f(x - 1) + 2$



(ii) (1) $y = -f(x) - 1$
(2) $y = f(2x) - 1$
(3) $y = \frac{1}{2}f(-2x)$

4.3 Equations and inequalities

Q20. (1) $-1, \frac{1}{3}$
(2) $-1, 2$
(3) $-\frac{7}{4}, \frac{3}{4}$
(4) $-\frac{8}{3}, \frac{4}{3}$
(5) 2
(6) -1
(7) 11
(8) $\frac{4}{3}$
(9) -1
(10) 1
(11) 3
(12) $\frac{1}{2}$

Q21. (i) $-2, 2$
(ii) $0, 4$

Q22. (i) -3
(ii) -5

Q23. (i) $\frac{1}{2}$
(ii) $\frac{7}{2}$

Q24. (i) 9
(ii) 4.5

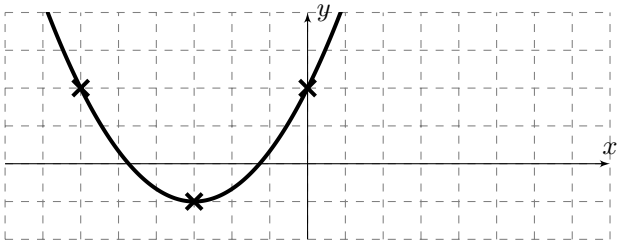
Q25. (1) 0: —
(2) 1: -1.31
(3) 2: $-1, 1.54$
(4) 1: 0.0605
(5) 3: $-0.0644, 3.17, 4.89$
(6) 1: 2.21
(7) 1: -1.52
(8) 3: $-0.481, 1.31, 3.17$
(9) 2: $-1.15, 1.15$
(10) 2: $-1.22, 0.549$
(11) 2: $0.780, 5.55$
(12) 2: $-5.24, -0.764$

Q26. (1) $x \leq -0.861$ or $0.746 \leq x \leq 3.11$
(2) $-4.59 < x < -0.887$ or $x > 1.47$
(3) $-0.535 \leq x \leq 0.444$ or $x \geq 3.69$
(4) $-1.65 < x < 1.27$ or $2 < x < 2.38$
(5) $-0.618 < x < 0$ or $1.62 < x \leq 2$
(6) $0 \leq x \leq 1$
(7) $-1.88 \leq x < -1$ or $0.347 \leq x \leq 1.53$
(8) $-2 \leq x \leq -1.41$ or $-1 < x \leq 1.41$
(9) $-3 < x \leq -2$ or $-1.41 \leq x < -1$ or $1.41 \leq x < 3$
(10) $-0.562 < x < 1$ or $3.56 < x \leq 4$
(11) $1 \leq x \leq 3.56$
(12) $-2.41 < x < -0.305$
(13) $0.918 \leq x \leq 2.66$

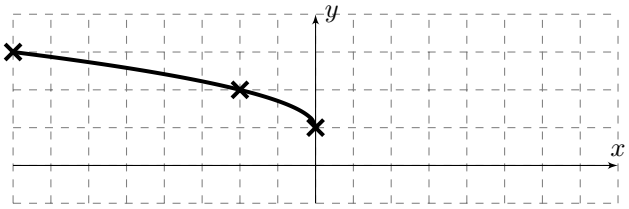
4.4 Chapter review

non-calculator questions

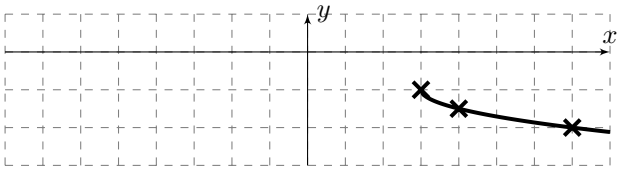
Q1. (1) vertical dilation by $\frac{1}{3}$
followed by
shift 3 left and 1 down



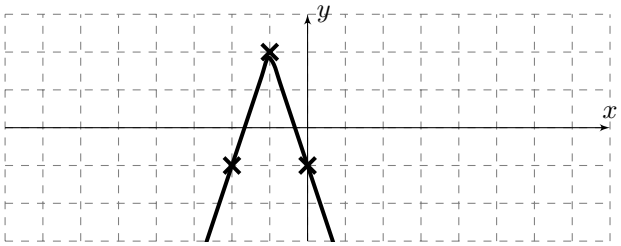
(2) horizontal dilation by -2
followed by
shift 1 up



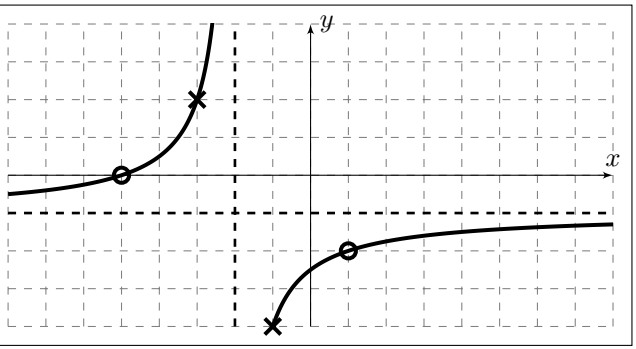
(3) vertical dilation by $-\frac{1}{2}$
followed by
shift 3 right and 1 down



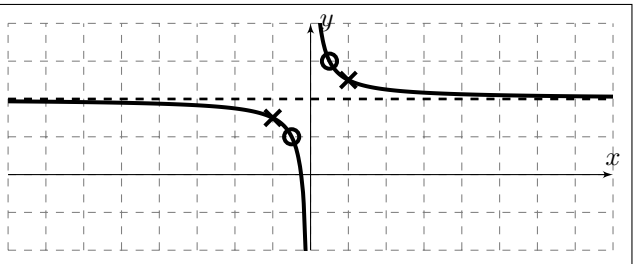
(4) vertical dilation by -3
followed by
shift 1 left and 2 up



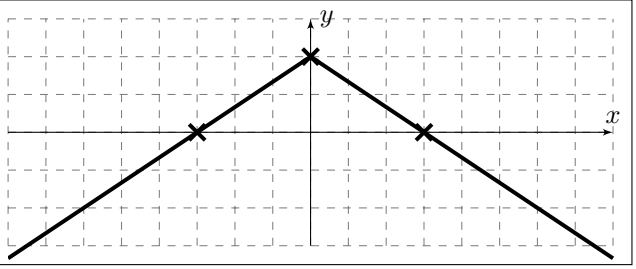
(5) vertical (or horizontal) dilation by -3
followed by
shift 2 left and 1 down



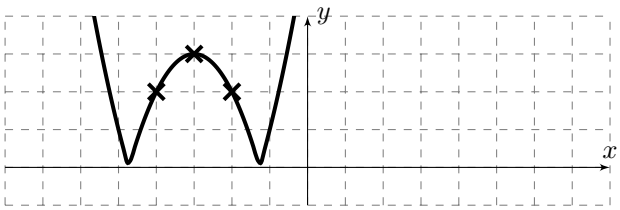
(6) vertical (or horizontal) dilation by $-\frac{1}{2}$
followed by
shift 2 up



(7) reflection of the part right of y-axis in the axis



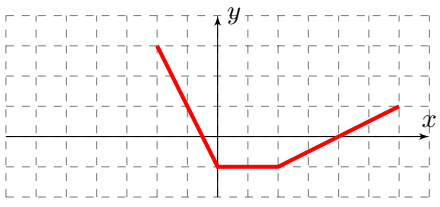
(8) shift 3 left and 3 down
followed by
reflection of the part below the x-axis in th axis



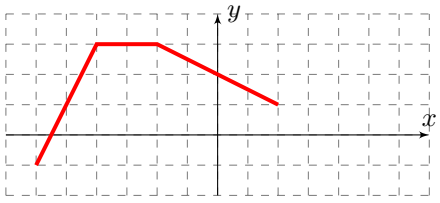
Q2. (1) $y = -2(x+3)^2 + 3$
(2) $y = \frac{-2}{x-1} - 1$
(3) $y = \frac{1}{2}(x-2)^3 + 1$
(4) $y = -\frac{2}{3}|x+3| + 2$
(5) $y = \sqrt{-2x} - 1$
(6) $y = ||x+3| - 2|$

Q3. (1) even (2) odd (3) neither

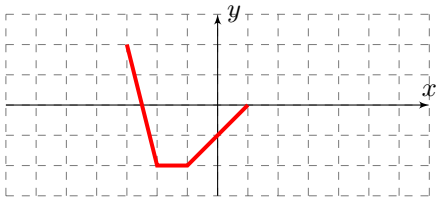
Q4. (i) (1)



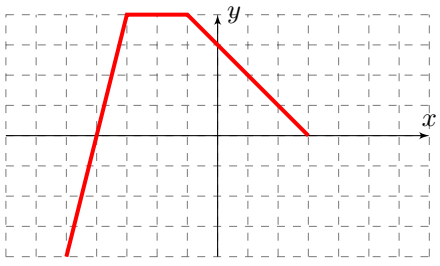
(2)



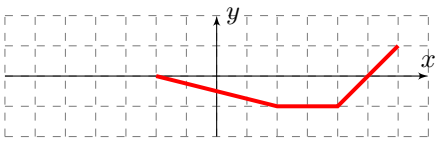
(3)



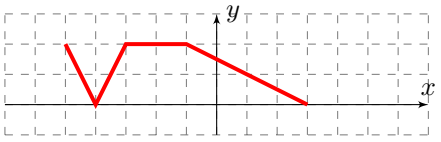
(4)



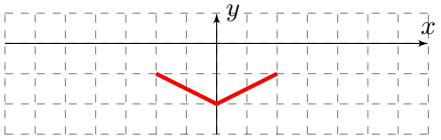
(5)



(6)



(7)



(ii) (1) $y = f(x-2) + 1$
(2) $y = -f(x+1) + 1$
(3) $y = -2f(x-1)$
(4) $y = \frac{1}{2}f(x) - 1$
(5) $y = |f(x) + 1|$
(6) $y = f(2x) + 1$
(7) $y = f(|x|)$

Q5. **DOMAIN**:
f_1: $x \le -\frac{3}{2}$
f_2: $x \in \mathbb{R}$
f_3: $x \in \mathbb{R}$
f_4: $x \in \mathbb{R}$
f_5: $x \in \mathbb{R}, x \neq -1$
f_6: $x \in \mathbb{R}$
f_7: $x > -3$
f_8: $x \le 3, x \neq -3$

RANGE:
f_1: $y \le 0$
f_2: $y \le 4$
f_3: $y \ge -1$
f_4: $y \ge -5$
f_5: $y \in \mathbb{R}, y \neq 3$
f_6: $y \le 2$

Q6.

function	(1)	(2)	(3)
domain	$[-5,-1[\cup]1,5]$	$]-5,4[$	$[-5,5]$
range	$[0,2]$	$]-4,2]$	$[-2,2]$
zeroes	$-4,4$	$-3,0,3$	$-4,0,4$
decreasing	$[-5,-4]$, $[2,4]$	$[-1,1]$	$[-5,-4]$, $[-2,2]$, $[4,5]$
increasing	$[-4,-2]$, $[4,5]$	$]-5,-1]$, $[1,4[$	$[-4,-2]$, $[2,4]$
constant	$[-2,-1[$, $]1,2]$	$\emptyset$	$\emptyset$
even	yes	no	no
odd	no	no	yes
one-to-one	no	no	no

calculator questions

Q7. (1) $(-3.59,-0.279)$, $(-0.549,-1.82)$, $(10.1,0.0986)$
(2) $(4.24,9.48)$, $(8.83,18.7)$
(3) $(-13.4,-0.590)$, $(0.561,20.1)$, $(15.3,-0.416)$
(4) $(-16.4,-6.82)$, $(0.382,0.159)$, $(16.0,6.66)$

Q8. (1) $x \in [-20,-3.59[\cup]-0.549,0[\cup]10.1,20]$ / $-20 \leq x < -3.59$ or $-0.549 < x < 0$ or $10.1 < x \leq 20$
(2) $x \in [-20,4.24[\cup]8.83,20]$ / $-20 \leq x < 4.24$ or $8.83 < x \leq 20$
(3) $x \in [-20,-13.4]\cup]0.5,0.561] \cup [15.3,20]$ / $-20 \leq x \leq -13.4$ or $0.5 < x \leq 0.561$ or $15.3 \leq x \leq 20$
(4) $x \in [-20,-16.4] \cup [0.382,16.0]$ / $-20 \leq x \leq -16.4$ or $0.382 \leq x \leq 20$
(answer can be given in any of the two forms shown above)

Chapter 5
Quadratics

5.1 Solving quadratic equations

Factorisation

Q1. (1) x^2+3x+2
(2) x^2+4x+3
(3) $x^2+7x+10$
(4) x^2-4x+3
(5) x^2-6x+8
(6) $x^2-13x+12$
(7) x^2+x-6
(8) x^2-x-6
(9) x^2+2x-8
(10) x^2-9
(11) x^2-x-12
(12) $x^2+4x-12$
(13) $x^2-2x-24$
(14) $x^2+5x-24$
(15) $x^2+4x-21$

Q2. (1) $2x^2+3x+1$
(2) $2x^2+9x+10$
(3) $2x^2-7x+3$
(4) $2x^2-5x+3$
(5) $2x^2-x-1$
(6) $3x^2+x-2$
(7) $3x^2-5x-2$
(8) $3x^2-x-2$
(9) $6x^2+13x-5$
(10) $6x^2-13x+5$
(11) $6x^2+7x-5$
(12) $15x^2-16x+4$
(13) $15x^2-17x-4$
(14) $15x^2+4x-4$
(15) $12x^2-25x+12$

Q3. (1) $(x)(x-2)$
(2) $(x+2)(x+1)$
(3) $(x+3)(x+2)$
(4) $(x+3)(x+1)$
(5) $(x+4)(x)$
(6) $(x+4)(x+1)$
(7) $(x+4)(x+2)$
(8) $(x+4)(x+3)$
(9) $(x+5)(x+1)$
(10) $(x+5)(x+2)$
(11) $(x+6)(x+2)$
(12) $(x+1)(x+12)$
(13) $(x-1)(x-3)$
(14) $(x-1)(x-2)$
(15) $(x-2)(x-3)$
(16) $(x-1)(x-6)$
(17) $(x-2)(x-4)$
(18) $(x-1)(x-8)$
(19) $(x-3)(x-4)$
(20) $(x-2)(x-6)$
(21) $(x-1)(x-12)$

Q4. (1) $(x+3)(x-2)$
(2) $(x+2)(x-3)$
(3) $(x+4)(x-2)$
(4) $(x+2)(x-4)$
(5) $(x+1)(x-8)$
(6) $(x+3)(x-3)$
(7) $(x+6)(x-1)$
(8) $(x+3)(x-4)$
(9) $(x+2)(x-6)$
(10) $(x+6)(x-2)$
(11) $(x+4)(x-3)$
(12) $(x+4)(x-6)$
(13) $(x+2)(x-12)$
(14) $(x+8)(x-3)$
(15) $(x+24)(x-1)$

Q5. (1) $(x+1)(x+2)$
(2) $(x+2)(x+3)$
(3) $(x+1)(x+3)$
(4) $(x+2)(x+4)$
(5) $(x+1)(x+4)$
(6) $(x+2)(x+3)$
(7) $(x+3)(x+5)$
(8) $(x+3)(x+4)$
(9) $(x+2)(x+6)$
(10) $(x-1)(x-2)$
(11) $(x-2)(x-3)$
(12) $(x-1)(x-8)$
(13) $(x-2)(x-4)$
(14) $(x-1)(x-4)$
(15) $(x-2)(x-2)$
(16) $(x-3)(x-5)$
(17) $(x-3)(x-4)$
(18) $(x-2)(x-6)$
(19) $(x+2)(x-4)$
(20) $(x+1)(x-8)$
(21) $(x+6)(x-2)$
(22) $(x+3)(x-4)$
(23) $(x-3)(x+5)$
(24) $(x+2)(x-12)$
(25) $(x-3)(x+8)$
(26) $(x+4)(x-6)$
(27) $(x-1)(x+24)$

Q6. (1) $(x+1)(2x+1)$
(2) $(2x+1)(x+2)$
(3) $(2x+3)(x+1)$
(4) $(2x+3)(x+2)$
(5) $(x+3)(2x+1)$
(6) $(2x+5)(x+2)$
(7) $(2x-1)(x-1)$
(8) $(2x-1)(x-2)$
(9) $(2x-1)(x-3)$
(10) $(2x-3)(x-1)$
(11) $(x+1)(2x-1)$
(12) $(2x+1)(x-1)$
(13) $(x+2)(3x-1)$
(14) $(x+1)(3x-2)$
(15) $(3x+1)(x-2)$
(16) $(3x+2)(x-1)$
(17) $(3x-4)(x-2)$
(18) $(3x-2)(x-4)$
(19) $(3x-1)(x-8)$
(20) $(3x+2)(x-4)$
(21) $(4x-3)(3x-4)$

Q7. (1) $(3x-1)(2x-1)$
(2) $(2x-1)(3x-2)$
(3) $(3x+2)(2x-1)$
(4) $(3x+1)(2x-3)$
(5) $(3x+2)(2x-3)$
(6) $(3x+2)(3x-1)$
(7) $(2x+5)(3x-1)$
(8) $(3x-5)(2x-1)$
(9) $(3x+5)(2x-1)$
(10) $(5x-1)(3x-4)$
(11) $(5x-2)(3x-2)$
(12) $(5x+1)(3x-4)$
(13) $(3x+2)(5x-2)$
(14) $(3x+5)(2x-1)$
(15) $(2x+5)(3x-1)$
(16) $(3x+1)(2x-5)$
(17) $(2x+1)(3x-5)$
(18) $(2x+3)(5x-3)$
(19) $(5x+9)(2x-1)$
(20) $(5x+1)(2x-9)$
(21) $(5x+2)(3x-1)$

Completing the square

Q8. (1) x^2-2x+3
(2) x^2-4x+0
(3) $2x^2+4x-2$
(4) $3x^2+24x+28$
(5) $-4x^2-4x+3$
(6) $5x^2-10x-5$
(7) $-\frac{1}{2}x^2+2x-\frac{7}{2}$
(8) $\frac{2}{3}x^2-\frac{8}{3}x+\frac{7}{6}$
(9) $\frac{1}{4}x^2+1x+3$

Q9. (1) $(x+1)^2+1$
(2) $(x+1)^2-1$
(3) $(x-2)^2-3$
(4) $(x+2)^2+1$
(5) $(x-3)^2+1$
(6) $(x+3)^2-2$
(7) $(x+4)^2-8$
(8) $(x-1.5)^2-5.25$
(9) $(x+2.5)^2-0.25$
(10) $2(x-1)^2$
(11) $2(x+1)^2-2$
(12) $2(x+2)^2-6$
(13) $2(x-2)^2+4$
(14) $2(x-1.5)^2-3.5$
(15) $\frac{1}{2}(x-1)^2+\frac{3}{2}$
(16) $-2(x+\frac{5}{2})^2+\frac{29}{2}$
(17) $-(x+2)^2+6$
(18) $-\frac{1}{2}(x-2)^2+1$
(19) $2(x+1)^2+4$
(20) $-2(x+\frac{1}{2})^2+\frac{1}{2}$
(21) $-\frac{1}{3}(x-\frac{3}{2})^2+\frac{5}{4}$

Q10. (1) (i) $(x+1)^2+2=0$ (ii) no solutions
(2) (i) $(x+2)^2-5=0$ (ii) $-2\pm\sqrt{5}$
(3) (i) $2(x+2)^2-5=0$ (ii) $-2\pm\frac{\sqrt{10}}{2}$
(4) (i) $3(x+1)^2-3=0$ (ii) $-2, 0$
(5) (i) $-4(x-1)^2+3=0$ (ii) $1\pm\frac{\sqrt{3}}{2}$
(6) (i) $-5(x+1)^2+9=0$ (ii) $-1\pm\frac{3\sqrt{5}}{5}$
(7) (i) $\frac{1}{3}(x+3)^2+\frac{1}{2}=0$ (ii) no solutions
(8) (i) $-\frac{2}{3}(x+3)^2+5=0$ (ii) $-3\pm\frac{\sqrt{30}}{2}$
(9) (i) $-\frac{3}{4}(x-\frac{2}{3})^2-\frac{2}{3}=0$ (ii) no solutions
(10) (i) $\frac{4}{3}(x+6)^2-10=0$ (ii) $-6\pm\frac{\sqrt{30}}{2}$
(11) (i) $-\frac{5}{3}(x-\frac{6}{5})^2+1=0$ (ii) $\frac{6}{5}\pm\frac{\sqrt{15}}{5}$
(12) (i) $\frac{2}{5}(x+2)^2-2=0$ (ii) $-2\pm\sqrt{5}$

Quadratic formula

Q11. (1) -2.62, -0.382
(2) no solution
(3) -7.16, -0.838
(4) 0.764, 5.24
(5) -3.41, -0.586
(6) -3.62, -1.38
(7) -1.24, 3.24
(8) -5.19, 0.193
(9) -5.16, 1.16
(10) -6.61, 0.606
(11) -1.47, 7.47
(12) -5.46, 1.46
(13) -0.851, 2.35
(14) 0.614, 4.89
(15) -3, 0.667
(16) no solution

(17) -5.26, 0.76
(18) -2.82, 1.07
(19) 2.2
(20) 1.16, 4.49
(21) -2.77, 1.44
(22) -2.26, 0.591
(23) -2
(24) no solution

Q12. (1) $\frac{-3-\sqrt{5}}{2}$, $\frac{-3+\sqrt{5}}{2}$
(2) no solution
(3) $-4-\sqrt{10}$, $-4+\sqrt{10}$
(4) $3-\sqrt{5}$, $3+\sqrt{5}$
(5) $-2-\sqrt{2}$, $-2+\sqrt{2}$
(6) $\frac{-5-\sqrt{5}}{2}$, $\frac{-5+\sqrt{5}}{2}$
(7) $\frac{-1}{2}$, 4
(8) -2, $\frac{3}{2}$
(9) $\frac{-3-\sqrt{23}}{2}$, $\frac{-3+\sqrt{23}}{2}$
(10) $\frac{3-\sqrt{31}}{2}$, $\frac{3+\sqrt{31}}{2}$
(11) $\frac{-2-2\sqrt{7}}{3}$, $\frac{-2+2\sqrt{7}}{3}$
(12) no solution
(13) no solution
(14) $\frac{1}{2}$, -6
(15) $\frac{2}{3}$, -3
(16) no solution
(17) $\frac{1}{2}$, -5
(18) $\frac{-7-\sqrt{65}}{8}$, $\frac{-7+\sqrt{65}}{8}$
(19) $\frac{-1-\sqrt{6}}{5}$, $\frac{-1+\sqrt{6}}{5}$
(20) $\frac{2-\sqrt{7}}{3}$, $\frac{2+\sqrt{7}}{3}$
(21) $\frac{-9-\sqrt{89}}{4}$, $\frac{-9+\sqrt{89}}{4}$
(22) $\frac{-5-\sqrt{37}}{6}$, $\frac{-5+\sqrt{37}}{6}$
(23) $\frac{-5-\sqrt{13}}{4}$, $\frac{-5+\sqrt{13}}{4}$
(24) $\frac{2-\sqrt{7}}{2}$, $\frac{2+\sqrt{7}}{2}$

Q13. -3, 0 **Q14.** $m < -6$ or $m > 6$ **Q15.** $m \neq 1$ **Q16.** $3 < m < 7$

5.2 Parabola

Q17. (1) x-int.: $(-2,0)$, $(-1,0)$, vertex: $(-1.5,-0.25)$, y-intercept: $(0,2)$
(2) x-int.: $(-1,0)$, $(3,0)$, vertex: $(1,-4)$, y-intercept: $(0,-3)$
(3) x-int.: $(-7.16,0)$, $(-0.838,0)$, vertex: $(-4,-10)$, y-intercept: $(0,6)$
(4) x-int.: none, vertex: $(1.5,3.75)$, y-intercept: $(0,6)$
(5) x-int.: $(-3.41,0)$, $(-0.586,0)$, vertex: $(-2,-2)$, y-intercept: $(0,2)$
(6) x-int.: $(-8.87,0)$, $(-1.13,0)$, vertex: $(-5,-7.5)$, y-intercept: $(0,5)$
(7) x-int.: $(-1,0)$, $(2,0)$, vertex: $(0.5,-4.5)$, y-intercept: $(0,-4)$
(8) x-int.: $(0.209,0)$, $(4.79,0)$, vertex: $(2.5,5.25)$, y-intercept: $(0,-1)$
(9) x-int.: $(-0.225,0)$, $(2.22,0)$, vertex: $(1,3)$, y-intercept: $(0,1)$
(10) x-int.: $(-1,0)$, $(4,0)$, vertex: $(1.5,6.25)$, y-intercept: $(0,4)$
(11) x-int.: $(-0.333,0)$, $(1,0)$, vertex: $(0.333,1.333)$, y-intercept: $(0,1)$
(12) x-int.: $(-0.667,0)$, $(0.5,0)$, vertex: $(-0.083,2.042)$, y-intercept: $(0,2)$

Q18. $y = -x^2 - 2x + 8$
Q19. $y = -2x^2 + 12x - 10$
Q20. $y = \frac{1}{2}x^2 - \frac{1}{2}x - 3$
Q21. $y = -2x^2 + 8x - \frac{7}{2}$
Q22. $y = -\frac{3}{2}x^2 + \frac{21}{2}x - 15$
Q23. $y = x^2 + 10x + 24$
Q24. $y = -\frac{3}{2}x^2 + 6x - 1$
Q25. $y = -1x^2 + 8x - 18$
Q26. $y = -\frac{3}{2}x^2 + 9x - \frac{25}{2}$
Q27. $y = -2x^2 - 4x + 6$
Q28. $y = x^2 - 2x - 1$
Q29. $y = 2x^2 + 12x + 17$
Q30. $y = -\frac{1}{2}x^2 - 2x + 3$
Q31. $y = (x+1)^2 - 4$, $y = (x+3)(x-1)$
Q32. $y = 2(x-\frac{3}{4})^2 - \frac{25}{8}$, $y = 2(x+\frac{1}{2})(x-2)$
Q33. $y = -3x^2 - 6x - \frac{9}{4}$, $y = -3(x+\frac{1}{2})(x+\frac{3}{2})$
Q34. $y = \frac{1}{2}x^2 - 2x$, $y = \frac{1}{2}(x)(x-4)$
Q35. $y = -\frac{1}{2}x^2 - 3x - 4$, $y = -\frac{1}{2}(x+3)^2 + \frac{1}{2}$
Q36. $y = -2x^2 - x + 3$, $y = -2(x+\frac{1}{4})^2 + 3\frac{1}{8}$

Q37.

	$y =$	$y =$	$y =$			
(1)	$x^2 - 2x - 15$	$(x-1)^2 - 16$	$(x-5)(x+3)$	$-3, 5$	$(1,-16)$	64
(2)	$2x^2 - 12x + 10$	$2(x-3)^2 - 8$	$2(x-1)(x-5)$	$1, 5$	$(3,-8)$	64
(3)	$2x^2 + 5x - 3$	$2(x+\frac{5}{4})^2 - \frac{49}{8}$	$2(x-\frac{1}{2})(x+3)$	$-3, \frac{1}{2}$	$(-\frac{5}{4}, -\frac{49}{8})$	49
(4)	$x^2 + 4x + 1$	$(x+2)^2 - 3$	$(x+2-\sqrt{3})(x+2+\sqrt{3})$	$-2 \pm \sqrt{3}$	$(-2,-3)$	12
(5)	$-2x^2 - 18x - 28$	$-2(x+\frac{9}{2})^2 + \frac{25}{2}$	$-2(x+2)(x+7)$	$-2, -7$	$(-\frac{9}{2}, \frac{25}{2})$	100
(6)	$-4x^2 + 28x - 49$	$-4(x-\frac{7}{2})^2$	$-4(x-\frac{7}{2})^2$	3.5	$(\frac{7}{2}, 0)$	0
(7)	$-\frac{1}{2}x^2 - 2x - 3$	$-\frac{1}{2}(x+2)^2 - 1$	—	none	$(-2,-1)$	-2
(8)	$-4x^2 + 8x - 2$	$-4(x-1)^2 + 2$	$-4(x-1+\frac{\sqrt{2}}{2})(x-1-\frac{\sqrt{2}}{2})$	$1 \pm \frac{\sqrt{2}}{2}$	$(1,2)$	32
(9)	$-2x^2 + 2x\sqrt{2} + 7$	$-2(x-\frac{\sqrt{2}}{2})^2 + 8$	$-2(x-\frac{\sqrt{2}}{2}+2)(x-\frac{\sqrt{2}}{2}-2)$	$\frac{\sqrt{2}}{2} \pm 2$	$(\frac{\sqrt{2}}{2}, 8)$	64

5.3 Applications of quadratics

Quadratic inequalities

Q38. (1) $]-\infty,-2]\cup[-1,+\infty[$
(2) $[-3,-\frac{1}{3}]$
(3) $]\frac{1}{2},2[$
(4) $]-\infty,-2[\cup]\frac{1}{2},+\infty[$
(5) $]-\infty,-1]\cup[-\frac{1}{3},+\infty[$
(6) $]-\infty,-\frac{7}{3}]\cup[1,+\infty[$
(7) $]-3,-\frac{1}{3}[$
(8) $]-\frac{1}{2},\frac{3}{2}[$
(9) $[-\frac{1}{2},1]$
(10) no solutions
(11) $\mathbb{R}$
(12) $]-\infty,-\frac{1}{2}[\cup]-\frac{1}{3},+\infty[$

Q39. (1) $]-\infty,-3.41[\cup]-0.586,+\infty[$
(2) $]-\infty,-3.28[\cup]0.61,+\infty[$
(3) $]-\infty,0.634]\cup[2.37,+\infty[$
(4) no solutions
(5) $\mathbb{R}$
(6) $\mathbb{R}$
(7) $[0.219,2.28]$
(8) $]-\infty,-0.897]\cup[2.23,+\infty[$
(9) $]-\infty,0.719[\cup]2.78,+\infty[$
(10) $]-2.57,1.07[$
(11) $[-2.23,0.897]$
(12) $]-\infty,-0.693]\cup[1.44,+\infty[$
(13) $]-12,-2.87[$
(14) $]-\infty,-1.43[\cup]0.904,+\infty[$
(15) $[-0.48,0.956]$
(16) $]-\infty,-2.26]\cup[0.591,+\infty[$
(17) $\mathbb{R}$
(18) no solutions

Problems involving quadratics

Q40. (i) 12 hours (ii) 6 hours (iii) 72 km

Q41. (i) 27 km (ii) 9 hours (iii) 4 hours (iv) 75 km

Q42. (i) 12 m (ii) 2.58 s (iii) 0.816 s (iv) 15.3 m

Q43. $50m \times 100m$

Q44. both 11; 121

Q45. 11 and 5.5; 60.5

Q46. 4 and 6; 24

Q47. 0.5

Q48. (i) $y = 6; 2\sqrt{10}$
(ii) $y = 8; 2\sqrt{11}$
(iii) $p = 2; 2\sqrt{5}$

Q49. $\frac{\sqrt{2}}{2}$

Q50. (i) no (ii) $3m4cm$ (iii) $2m26cm$

Q51. $3.01m$

Q52. (i) $2400m$
(ii) $5100m$
(iii) $5420m$
(iv) 14 hrs 20 mins
(v) 23 hrs 20 mins

Q53. $6.5m \times 13m$

Q54. $0.75m^2 = 7500cm^2$

Q55. $9mm$

Q56. $20.7cm$ or $54.3cm$

Q57. $85cm$

Q58. yes: edge 1m long veritcally, edge 85cm long across the ditch, edge 2m long along the ditch

Investigating graphs of rational functions

Q59. (1) **1.** —
2. $(1,0)$
3. $(0,\frac{1}{2})$
4. $x = 2$
5. $y = 1$
6. —

(2) **1.** —
2. $(-2,0)$
3. $(0,-4)$
4. $x = 1$
5. $y = 2$
6. —

(3) **1.** —
2. $(3,0)$
3. $(0,-3)$
4. $x = -2$
5. $y = 2$
6. —

(4) **1.** —
2. $(2,0)$
3. $(0,-\frac{1}{2})$
4. $x = -2$
5. $y = \frac{1}{2}$
6. —

(5) **1.** —
2. $(-1,0)$
3. $(0,-\frac{1}{3})$
4. $x = 1$
5. $y = \frac{1}{3}$
6. —

(6) **1.** $y = 2$
2. none
3. $(0,2)$
4. none
5. none
6. —

(7) **1.** $y = \frac{1}{x+3}$
2. none
3. $(0, \frac{1}{3})$
4. $x = -3$
5. $y = 0$
6. —

(8) **1.** $y = \frac{1}{x+3}$
2. none
3. $(0, \frac{1}{3})$
4. $x = -3$
5. $y = 0$
6. —

(9) **1.** $y = \frac{1}{x-2}$
2. none
3. $(0, -\frac{1}{2})$
4. $x = 2$
5. $y = 0$
6. —

(10) **1.** $y = \frac{x+2}{x-2}$
2. $(-2, 0)$
3. $(0, -1)$
4. $x = 2$
5. $y = 1$
6. —

(11) **1.** $y = \frac{x+3}{x+2}$
2. $(-3, 0)$
3. $(0, 1.5)$
4. $x = -2$
5. $y = 1$
6. —

(12) $y = \frac{2x+1}{x-1}$
1. —
2. none
3. $(0, -1)$
4. $x = 1$
5. $y = 2$
6. —

(13) **1.** $y = \frac{(x+3)(x-2)}{(x-3)(x+2)}$
2. $(-3, 0), (2, 0)$
3. $(0, 1)$
4. $x = -2, x = 3$
5. $y = 1$
6. —

(14) **1.** $y = \frac{(x-3)(x-2)}{(x+3)(x+2)}$
2. $(2, 0), (3, 0)$
3. $(0, 1)$
4. $x = -3, x = -2$
5. $y = 1$
6. —

(15) **1.** $y = \frac{(x+3)(2x-1)}{(x-3)(x+1)}$
2. $(-3, 0), (\frac{1}{2}, 0)$
3. $(0, 1)$
4. $x = -1, x = 3$
5. $y = 2$
6. —

(16) **1.** $y = \frac{2(x-1)(x+2)}{(x-2)(x+3)}$
2. $(-2, 0), (1, 0)$
3. $(0, \frac{2}{3})$
4. $x = -3, x = 2$
5. $y = 2$
6. —

(17) **1.** $y = \frac{(x+3)(2x-5)}{(x-5)(2x+3)}$
2. $(-3, 0), (2.5, 0)$
3. $(0, 1)$
4. $x = -1.5, x = 5$
5. $y = 1$
6. —

(18) **1.** $y = \frac{x^2-4}{x+1}$
2. $(\pm 2, 0)$
3. $(0, -4)$
4. $x = -1$
5. none
6. —

Chapter 6

Trigonometry

6.1 Degrees and radians

Q1. (1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$ (5) $\frac{\pi}{12}$ (6) $\frac{3\pi}{4}$ (7) $\frac{2\pi}{3}$ (8) $\frac{3\pi}{2}$ (9) $\frac{\pi}{9}$ (10) $\frac{5\pi}{18}$ (11) $\frac{5\pi}{12}$ (12) $\frac{11\pi}{6}$ (13) $\frac{7\pi}{12}$ (14) $\frac{7\pi}{6}$ (15) $\frac{5\pi}{6}$

Q2. (1) 3.14 (2) 1.57 (3) 1.40 (4) 0.209 (5) 1.75 (6) 0.995 (7) 1.36 (8) 1.88 (9) 3.49 (10) 0.314 (11) 1.26 (12) 5.10

Q3. (1) 30° (2) 120° (3) 225° (4) 22.5° (5) 75° (6) 105° (7) 300° (8) 40° (9) 100° (10) 150° (11) 172° (12) 90.0° (13) 57.3° (14) 50.0° (15) 69.9°

6.2 Trigonometric ratios

Q4.

θ	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Q5.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
40°	0.643	0.766	0.839
60°	0.866	0.5	1.73
1°	0.0175	1	0.0175
1	0.841	0.54	1.56
13.5°	0.233	0.972	0.24
1.5	0.997	0.0707	14.1
0.8	0.717	0.697	1.03
8°	0.139	0.99	0.141
1.57	1	0.000796	1260
0.3	0.296	0.955	0.309
1.2	0.932	0.362	2.57
1.2°	0.0209	1	0.0209

Q6. (1) $\sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}, \tan\alpha = \frac{3}{4}$
(2) $\sin\alpha = \frac{4}{5}, \cos\alpha = \frac{3}{5}, \tan\alpha = \frac{4}{3}$
(3) $\sin\alpha = \frac{5}{13}, \cos\alpha = \frac{12}{13}, \tan\alpha = \frac{5}{12}$
(4) $\sin\alpha = \frac{5}{\sqrt{41}}, \cos\alpha = \frac{4}{\sqrt{41}}, \tan\alpha = \frac{5}{4}$
(5) $\sin\alpha = \frac{3}{\sqrt{10}}, \cos\alpha = \frac{1}{\sqrt{10}}, \tan\alpha = 3$
(6) $\sin\alpha = \frac{\sqrt{5}}{3}, \cos\alpha = \frac{2}{3}, \tan\alpha = \frac{\sqrt{5}}{2}$
(7) $\sin\alpha = \frac{15}{17}, \cos\alpha = \frac{8}{17}, \tan\alpha = \frac{15}{8}$
(8) $\sin\alpha = \frac{\sqrt{7}}{4}, \cos\alpha = \frac{3}{4}, \tan\alpha = \frac{\sqrt{7}}{3}$
(9) $\sin\alpha = \frac{\sqrt{11}}{6}, \cos\alpha = \frac{5}{6}, \tan\alpha = \frac{\sqrt{11}}{5}$

Q7. (1) $\cos\alpha = \frac{4}{5}$, $\tan\alpha = \frac{3}{4}$
(2) $\cos\alpha = \frac{2}{\sqrt{13}}$, $\tan\alpha = \frac{3}{2}$
(3) $\sin\alpha = \frac{3}{\sqrt{21}}$, $\tan\alpha = \frac{\sqrt{3}}{2}$
(4) $\sin\alpha = \frac{9}{41}$, $\tan\alpha = \frac{9}{40}$
(5) $\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$
(6) $\sin\alpha = \frac{1}{\sqrt{2}}$, $\cos\alpha = \frac{1}{\sqrt{2}}$
(7) $\cos\alpha = \frac{1}{\sqrt{5}}$, $\tan\alpha = 2$
(8) $\sin\alpha = \frac{1}{3}$, $\tan\alpha = \frac{1}{2\sqrt{2}}$
(9) $\sin\alpha = \frac{2}{3}$, $\cos\alpha = \frac{\sqrt{5}}{3}$
(10) $\cos\alpha = \frac{3\sqrt{2}}{5}$, $\tan\alpha = \frac{\sqrt{14}}{6}$
(11) $\sin\alpha = \frac{7}{8}$, $\tan\alpha = \frac{7}{\sqrt{15}}$
(12) $\sin\alpha = \frac{5}{11}$, $\cos\alpha = \frac{4\sqrt{6}}{11}$

Q8. (1) 15 (2) 12 (3) 3.4 (4) 4.5 (5) 7.5 (6) $\sqrt{29}$ (7) $6\sqrt{10}$ (8) $6\sqrt{5}$ (9) 12.5

Q9. (1) 0.748 or 42.8° (2) 1.44 or 82.2° (3) 0.779 or 44.6° (4) 0.308 or 17.6° (5) 1.55 or 88.6° (6) 1.19 or 68° (7) 0.89 or 51° (8) no such angle (9) 1.25 or 71.6° (10) 0.527 or 30.2° (11) 0.503 or 28.8° (12) 0.202 or 11.6°

Q10. 22.9cm, 23.9cm **Q11.** 4.10cm, 9.35cm **Q12.** 3.77cm, 4.60cm **Q13.** 4.35cm, 4.83cm

Q14. (1) 8.75 (2) 23.3 (3) 1.98 (4) 0.0463 (5) 2.08 (6) 95.8 (7) 20.5 (8) 41.8 (9) 26.2 (10) 25.7 (11) 28.7 (12) 71.8 (13) 5.6 (14) 13.3 (15) 15.1 (16) 159 (17) 1.82 (18) 27.6

Q15. 205
Q16. 36.5
Q17. 23.0 m
Q18. 28.2 m
Q19. $9\sqrt{3} \approx 15.6$
Q20. 8.10
Q21. 12.4
Q22. 84.3
Q23. 4.7 m
Q24. 4.62 m
Q25. 12.8

6.3 Trigonometric functions

Q26.

	$\sin A$	$\cos A$	$\tan A$
(1)	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
(2)	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
(3)	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
(4)	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
(5)	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
(6)	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
(7)	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
(8)	-1	0	none
(9)	1	0	none
(10)	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
(11)	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
(12)	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
(13)	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
(14)	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
(15)	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
(16)	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$

(17)	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	(29)	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
(18)	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	(30)	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
(19)	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	(31)	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
(20)	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	(32)	0	1	0
(21)	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	(33)	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
(22)	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	(34)	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
(23)	0	-1	0	(35)	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
(24)	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	(36)	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
(25)	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	(37)	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
(26)	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	(38)	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
(27)	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	(39)	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
(28)	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	(40)	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$

Q27. (1) $\cos\alpha = -\frac{4}{5}, \tan\alpha = -\frac{3}{4}$
(2) $\cos\alpha = -\frac{2}{\sqrt{13}}, \tan\alpha = \frac{3}{2}$
(3) $\sin\alpha = -\frac{\sqrt{21}}{7}, \tan\alpha = -\frac{\sqrt{3}}{2}$
(4) $\sin\alpha = -\frac{9}{41}, \tan\alpha = \frac{9}{40}$
(5) $\sin\alpha = \frac{12}{13}, \cos\alpha = -\frac{5}{13}$
(6) $\sin\alpha = -\frac{1}{\sqrt{2}}, \cos\alpha = \frac{1}{\sqrt{2}}$
(7) $\cos\alpha = \frac{1}{\sqrt{5}}, \tan\alpha = -2$
(8) $\sin\alpha = \frac{1}{3}, \tan\alpha = -\frac{1}{2\sqrt{2}}$
(9) $\sin\alpha = -\frac{2}{3}, \cos\alpha = -\frac{\sqrt{5}}{3}$
(10) $\cos\alpha = \frac{3\sqrt{2}}{5}, \tan\alpha = -\frac{\sqrt{14}}{6}$
(11) $\sin\alpha = -\frac{7}{8}, \tan\alpha = \frac{7}{\sqrt{15}}$
(12) $\sin\alpha = \frac{5}{11}, \cos\alpha = -\frac{4\sqrt{6}}{11}$

Q28. (1) $\frac{\pi}{2}$ or $90°$
(2) $-\frac{\pi}{2}$ or $-90°$
(3) 0
(4) $-\frac{3\pi}{4}$ or $-135°$
(5) $-\frac{\pi}{6}$ or $-30°$
(6) $-\frac{5\pi}{6}$ or $-150°$
(7) $\frac{5\pi}{6}$ or $150°$
(8) $\frac{3\pi}{4}$ or $135°$
(9) $\frac{\pi}{3}$ or $60°$
(10) $-\frac{\pi}{3}$ or $-60°$
(11) $-\frac{\pi}{4}$ or $-45°$
(12) $-\frac{2\pi}{3}$ or $-120°$
(13) $\frac{2\pi}{3}$ or $120°$
(14) π or $180°$

Q29. (1) 2.42 or 139°
(2) −0.561 or −32.2°
(3) −2.84 or −163°
(4) −2.3 or −132°
(5) −2 or −115°
(6) −0.114 or −6.55°/−6.54°
(7) 0.326/0.325 or 18.7°/18.6°
(8) 0.403 or 23.1°
(9) 2.65 or 152°
(10) −0.263 or −15°
(11) 0.374 or 21.4°
(12) 2.44 or 140°
(13) −2.67 or −153°
(14) 0.61 or 34.9°
(15) −0.69 or −39.5°
(16) 0.543 or 31.1°
(17) 0.307 or 17.6°
(18) −1.38 or −78.8°
(19) −2.07 or −119°
(20) −0.767 or −43.9°
(21) −2.78 or −159°
(22) −1.88 or −107°
(23) −0.46 or −26.4°
(24) 2.45 or 140°

6.4 Trigonometric equations

Q30. (1) $\frac{\pi}{6}, \frac{5\pi}{6}$
(2) $\frac{\pi}{3}, \frac{5\pi}{3}$
(3) $\frac{\pi}{2}, -\frac{3\pi}{2}$
(4) $\pi, 3\pi$
(5) $\frac{\pi}{4}, \frac{3\pi}{4}$
(6) $\frac{\pi}{6}, \frac{11\pi}{6}$
(7) $\frac{7\pi}{6}, \frac{11\pi}{6}$
(8) $\frac{2\pi}{3}, \frac{4\pi}{3}$
(9) $\frac{5\pi}{4}, \frac{7\pi}{4}$
(10) $\frac{5\pi}{6}, \frac{7\pi}{6}$
(11) $\frac{-5\pi}{6}, \frac{-\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
(12) $\frac{-4\pi}{3}, \frac{-2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$
(13) $\frac{\pi}{3}, \frac{2\pi}{3}$
(14) $\frac{\pi}{4}, \frac{7\pi}{4}$
(15) $\frac{4\pi}{3}, \frac{5\pi}{3}$
(16) $\frac{3\pi}{4}, \frac{5\pi}{4}$
(17) $\frac{-11\pi}{3}, \frac{-10\pi}{3}, \frac{-5\pi}{3}, \frac{-4\pi}{3}$
(18) $\frac{-15\pi}{4}, \frac{-9\pi}{4}, \frac{-7\pi}{4}, \frac{-\pi}{4}$

Q31. (1) $\frac{\pi}{4}$
(2) $\frac{\pi}{3}$
(3) $\frac{\pi}{6}$
(4) $0, \pi$
(5) $\frac{3\pi}{4}$
(6) $\frac{2\pi}{3}$
(7) $\frac{5\pi}{6}$
(8) $-\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}$
(9) $\frac{-5\pi}{3}, \frac{-2\pi}{3}, \frac{\pi}{3}$
(10) $\frac{-11\pi}{6}, \frac{-5\pi}{6}, \frac{\pi}{6}$
(11) $\frac{-5\pi}{4}, \frac{-\pi}{4}, \frac{3\pi}{4}$

Q32. (1) $30°, 150°$
(2) $120°, 240°$
(3) $45°, 225°$
(4) $240°, 300°$
(5) $30°, 330°$
(6) $120°, 300°$
(7) $90°$
(8) $180°$
(9) $0°, 180°, 360°$
(10) $0°, 180°, 360°$
(11) $60°, 300°$
(12) $150°, 330°$
(13) no solutions
(14) $90°, 270°$
(15) $120°, 300°$
(16) $270°$
(17) $150°, 210°$
(18) $30°, 210°$

Q33. (1) $0.412, 2.73$
(2) $1.23, 5.05$
(3) $0.433, 2.71$
(4) $0.635, 5.65$
(5) $3.66, 5.76$
(6) $1.88, 4.41$
(7) $3.99, 5.44$
(8) $1.85, 4.43$
(9) $0.789, 2.35, 7.07, 8.64$
(10) $1.17, 5.11, 7.45, 11.4$
(11) $-5.6, -3.82, 0.682, 2.46$
(12) $-5.73, -0.555, 0.555, 5.73$
(13) $3.59, 5.84, 9.87, 12.1$
(14) $1.79, 4.49, 8.08, 10.8$
(15) $-2.74, -0.401, 3.54, 5.88$
(16) $-3.77, -2.51, 2.51, 3.77$

Q34. (1) 1.11
(2) 1.15
(3) 0.464
(4) 1.89
(5) 2.85
(6) 1.91
(7) 1.75
(8) $-5.03, -1.89, 1.25$
(9) $-4.95, -1.8, 1.34$
(10) $-5.7, -2.55, 0.588$
(11) $-4.14, -0.998, 2.14$

Q35. (1) $11.5°, 168°$
(2) $72.5°, 287°$
(3) $42°, 222°$
(4) $204°, 336°$
(5) $139°, 221°$
(6) $108°, 288°$
(7) $26.1°, 154°$
(8) $-76.7°, 76.7°$
(9) $-120°, 59.5°$
(10) $-169°, -11°$
(11) $-112°, 112°$
(12) $-66.5°, 113°$

Q36. (1) 0, π, 2π, $\frac{\pi}{6}$, $\frac{5\pi}{6}$
(2) $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, $\frac{11\pi}{6}$
(3) $\frac{\pi}{2}$, $\frac{3\pi}{2}$
(4) $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$
(5) $\frac{7\pi}{6}$, $\frac{11\pi}{6}$, 0.412, 2.73
(6) $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{3\pi}{2}$
(7) 0.34, 2.8, 5.55, 3.87
(8) $\frac{\pi}{6}$, $\frac{5\pi}{6}$, 0.73, 2.41
(9) $\frac{7\pi}{6}$, $\frac{11\pi}{6}$
(10) $\frac{\pi}{3}$, $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{5\pi}{3}$
(11) $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$
(12) 0, π, 2π
(13) $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, $\frac{11\pi}{6}$
(14) $\frac{\pi}{3}$, $\frac{5\pi}{3}$, 1.98, 4.3
(15) 0, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, 2π
(16) 1.91, 0.841, 4.37, 5.44
(17) $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, 2.3, 3.98
(18) $\frac{\pi}{3}$, $\frac{5\pi}{3}$
(19) 0, $\frac{\pi}{4}$, π, $\frac{5\pi}{4}$
(20) $\frac{\pi}{4}$, $\frac{3\pi}{4}$, $\frac{5\pi}{4}$, $\frac{7\pi}{4}$
(21) $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{4\pi}{3}$, $\frac{5\pi}{3}$
(22) $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, $\frac{11\pi}{6}$
(23) $\frac{3\pi}{4}$, $\frac{7\pi}{4}$, 1.11, 4.25
(24) $\frac{\pi}{4}$, $\frac{5\pi}{4}$, 0.464, 3.61
(25) $\frac{3\pi}{4}$, $\frac{7\pi}{4}$, 1.89, 5.03

6.5 Trigonometry in geometry

Q37. (1) $b \approx 5.53$, $c \approx 7.04$
(2) $b \approx 4.89$, $c \approx 6.63$
(3) $a \approx 8.96$, $c \approx 10.5$
(4) $c \approx 8.5$, $b \approx 4.14$
(5) $a \approx 15.1$, $b \approx 5.76$
(6) $c \approx 6.96$, $a \approx 0.625$
(7) $b \approx 4.31$, $a \approx 11.6$
(8) $a \approx 4.52$, $c \approx 2.82$
(9) $a \approx 3.19$, $b \approx 6.36$
(10) $c \approx 2.68$, $a \approx 4.41$
(11) $b \approx 2.22$, $a \approx 4.42$
(12) $b \approx 11.3$, $c \approx 1.21$
(13) $b \approx 14.1$, $a \approx 16.9$
(14) $a \approx 6.67$, $c \approx 11.6$
(15) $c \approx 9.66$, $b \approx 12.6$
(16) $a \approx 4.7$, $c \approx 3.69$
(17) $a \approx 12.1$, $b \approx 4.67$
(18) $b \approx 13.7$, $c \approx 13$
(19) $b \approx 2.09$, $a \approx 4.94$
(20) $c \approx 11.1$, $a \approx 10.8$

Q38. (1) $B \approx 49.9°$, $C \approx 85.1°$ or $B \approx 130°$, $C \approx 4.92°$
(2) $B \approx 61.1°$, $C \approx 61.9°$ or $B \approx 119°$, $C \approx 4.11°$
(3) $A \approx 6.28°$, $C \approx 161°$
(4) $C \approx 13.4°$, $B \approx 155°$ or $C \approx 167°$, $B \approx 1.37°$
(5) no such triangle
(6) $C \approx 51.8°$, $A \approx 74.2°$
(7) $B \approx 36°$, $A \approx 83°$
(8) $A \approx 57°$, $C \approx 78°$ or $A \approx 123°$, $C \approx 12°$
(9) no such triangle
(10) $C \approx 67.1°$, $A \approx 36.9°$
(11) no such triangle
(12) $B \approx 66.1°$, $C \approx 41.9°$
(13) $B \approx 49.1°$, $A \approx 86.9°$ or $B \approx 131°$, $A \approx 5.07°$
(14) $A \approx 29.4°$, $C \approx 109°$
(15) $C \approx 56.9°$, $B \approx 82.1°$ or $C \approx 123°$, $B \approx 15.9°$
(16) $A \approx 79.5°$, $C \approx 26.5°$ or $A \approx 101°$, $C \approx 5.45°$
(17) $A \approx 31.9°$, $B \approx 117°$ or $A \approx 148°$, $B \approx 0.924°$
(18) $B \approx 67.4°$, $C \approx 57.6°$ or $B \approx 113°$, $C \approx 12.4°$
(19) $B \approx 14.7°$, $A \approx 142°$
(20) $C \approx 60.9°$, $A \approx 62.1°$ or $C \approx 119°$, $A \approx 3.88°$

Q39. (1) $c \approx 6.88$ (2) $a \approx 3.14$ (3) $c \approx 3.2$ (4) $b \approx 4.19$ (5) $b \approx 5.56$ (6) $a \approx 2.29$ (7) $a \approx 6.6$ (8) $c \approx 7.77$ (9) $b \approx 5.47$ (10) $a \approx 8.82$ (11) $a \approx 4.7$ (12) $c \approx 2.22$ (13) $a \approx 1.88$ (14) $c \approx 4.9$ (15) $b \approx 2.94$ (16) $c \approx 7.34$ (17) $b \approx 3.89$ (18) $c \approx 9.33$ (19) $a \approx 1.75$ (20) $b \approx 8.53$

Q40. (1) $A \approx 46°$ (2) $C \approx 61.8°$ (3) $A \approx 87.1°$ (4) $B \approx 88.1°$ (5) $B \approx 22.5°$ (6) $C \approx 18.6°$ (7) $A \approx 47.5°$ (8) $C \approx 57.5°$ (9) $B \approx 117°$ (10) $C \approx 17.6°$ (11) $A \approx 14.6°$ (12) $B \approx 12.1°$ (13) $C \approx 24.1°$ (14) $B \approx 54.8°$ (15) $A \approx 59.8°$ (16) $B \approx 51°$ (17) $A \approx 25.8°$ (18) $C \approx 2.1°$ (19) $A \approx 65.2°$ (20) $B \approx 23.2°$

Q41. (1) $c \approx 3.75$ or $c \approx 26$ (2) $a \approx 17.9$ (3) no such triangle (4) $b \approx 6.32$ or $b \approx 12.4$ (5) $b \approx 0.984$ or $b \approx 11.1$ (6) $a \approx 1.5$ or $a \approx 28.3$ (7) no such triangle (8) $c \approx 11.4$ (9) $b \approx 5.78$ or $b \approx 29.9$ (10) $a \approx 9.16$ or $a \approx 23.1$ (11) $a \approx 30.6$ (12) $c \approx 1.06$ or $c \approx 6.56$ (13) $a \approx 3.26$ or $a \approx 6.94$ (14) no such triangle (15) $b \approx 11.9$ (16) $c \approx 11.8$ or $c \approx 24.3$ (17) $b \approx 14.1$ (18) $c \approx 2.41$ or $c \approx 4.8$ (19) $a \approx 14.8$ or $a \approx 46.1$ (20) $b \approx 1.3$ or $b \approx 5.64$

Q42. (i) $\frac{25}{4}(1+\sqrt{3}) \approx 17.1$ (ii) $\frac{5}{2}(4+2\sqrt{3}+\sqrt{6}-\sqrt{2}) \approx 21.2$

Q43. (i) 9 (ii) $9\sqrt{6}-3\sqrt{2} \approx 17.8$

Q44. 146°

Q45. 6.47

Q46. (i) 199° (ii) 019° (iii) 75.2 km (iv) 044° (v) 274°

Q47. 45.0 km, 85.1 km

Q48. (i) 56.4 km (ii) 55.4 km (iii) 72.8 km

Q49. 20 km or 52.3 km

Q50. (1) both 66.5 km (2) 60.9 km and 30.5 km

6.6 Arcs, sectors, segments

Q51. (1) 10.2 (2) 18.3 (3) 6.08 (4) 16.6 (5) 20.6 (6) 11.9 (7) 5.06 (8) 15.3

Q52. (1) 1.11 (2) 2.15 (3) 40.6 (4) 18.4 (5) 0.117 (6) 0.301

Q53. (1) 3.05 (2) 50.4 (3) 1.53 (4) 0.285 (5) 9.88 (6) 0.622

Q54. (1) 100° (2) 66.2° (3) 101° (4) 66.7° (5) 43.5° (6) 36°

Q55. $\frac{2}{3}$ rad or 38.2°

Q56. 57.1

Q57. (i) 12.5 (ii) 11.6

Q58. 18.00l

Q59. $50(\pi-2)cm^2 \approx 57.1cm^2$

Q60. $100(\frac{2\pi}{3}+\sqrt{3}-3)cm^2 \approx \approx 82.6cm^2$

Q61. (1) 10.4 (2) $19\pi \approx 59.7$ (3) 72.1 (4) 59.9

6.7 Chapter review

Q1. $x \in \{-\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{2}, \frac{4\pi}{3}, \frac{5\pi}{3}\}$

Q2. $x \in \{60°, 120°\}$

Q3. $x \in \{0.730, 2.41\}$

Q4. $x \in \{-180°, -60°, 60°, 180°, 300°\}$

Q5. $x \in \{-2\pi, -\frac{4\pi}{3}, -\frac{2\pi}{3}, 0, \frac{2\pi}{3}\}$

Q6. $x \in \{1.82, \pi, 4.46\}$

Q7. $x \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

Q8. $x \in \{1.38, \frac{5\pi}{6}, 4.52, \frac{11\pi}{6}\}$

Q9. $x \in \{-\frac{5\pi}{6}, -\frac{\pi}{6}\}$

Q10. $x \in \{-300°, -246°, -114°, -60°\}$

Q11. (i) $a = -2$ (ii) **(*)** $b = \frac{\pi}{3}$

Q12. (1) $\cos A = -\frac{\sqrt{5}}{3}$, $\tan A = -\frac{2}{\sqrt{5}}$
(2) $\cos A = \frac{5}{6}$, $\tan A = -\frac{\sqrt{11}}{5}$
(3) $\sin A = -\frac{2\sqrt{2}}{3}$, $\tan A = -2\sqrt{2}$
(4) $\sin A = \frac{\sqrt{21}}{5}$, $\tan A = -\frac{\sqrt{21}}{2}$
(5) $\sin A = -\frac{\sqrt{6}}{3}$, $\cos A = -\frac{1}{\sqrt{3}}$
(6) $\sin A = \frac{\sqrt{6}}{3}$, $\cos A = -\frac{1}{\sqrt{3}}$

Q13. (i) $6\pi - 4\sqrt{3} - 4\sqrt{2} - 4$
(ii) $8 + 3\pi$
(iii) 18.6

Q14. (i) $r \approx 6.71$
(ii) 19.3
(iii) 11.5

Q15. (i) $107m$
(ii) $266m$
(iii) $251000m^3$

Q16. (i) $855m$
(ii) $1880m$ (3 s.f.)

Q17. $AB \approx 6.40$, $AC \approx 7.14$

Q18. 39.3°, 46.7°

Q19. 73.7°, 53.1°, 53.1°

Q20. (i) $8.10km$
(ii) 101°

Chapter 7
Statistics

7.1 Types of data

Q1. (1) qualitative
(2) quantitative
(3) quantitative
(4) quantitative
(5) quantitative
(6) qualitative
(7) quantitative
(8) quantitative
(9) qualitative
(10) quantitative
(11) quantitative
(12) quantitative
(13) quantitative

Q2. (1) discrete
(2) continuous(?)
(3) discrete
(4) continuous(?)
(5) continuous(?)
(6) continuous
(7) continuous
(8) continuous
(9) continuous
(10) continuous
(11) discrete
(12) continuous
(13) discrete
(14) discrete
(15) continuous
(16) discrete
(17) continuous

7.2 Averages, range, quartiles

Q3.

	mean	mode	median	Q_1	Q_2	range	I.Q.R.
(1)	142	141	141	141	143	5	2
(2)	1(0.9997)	0.995	1	0.995	1.003	0.01	0.008
(3)	8.5	7	8.5	7	9.5	3.5	2.5
(4)	1000(999.75)	998	999.5	998	1001	5	3
(5)	3.25	4	3.5	2	4	5	2
(6)	83.4(83.375)	90	84.5	79.5	88.5	18	9
(7)	1.82	3	2	0	3	3	3
(8)	3.1	3	3	2	4	6	2

Q4. (1) (7) (2) (3) (3) (1) (4) (5) (5) (8) (6) (2) (7) (4) (8) (6)

Q5. 169cm **Q6.** 61.6kg **Q7.** 187.2cm **Q8.** 169cm **Q9.** 64.1kg

Q10. (1) 55.5 (2) 56.5 (3) 46.7

Q11. 2 min 22 sec **Q12.** 161.4cm **Q13.** 71.3kg

Q14. (1) 172.5 (2) 170.5 (3) 22

Q15. (1) mean: 73.8, mode: none, median: 84; mean takes all data into consideration, median is not affected by the "outliers"

(2) mean: 46.6, mode: 25, median: 22; median best, mode reasonable, too

(3) mode: trainers (the only)

Q16. (1) 70 (2) 65 (3) 79

Q17. 164, 178

Q18. (1) $a = 3, b = 5$

(2) $a = 7, b = 8$

(3) $a = 3, b = 9$

(4) $a = 3, b = 6$ or $a = 4, b = 5$

(5) $a = 13, b = 14, c = 16, d = 17, e = 19$

Q19. mode = 1.99zł, median = 2.05zł, mean = 2.14zł

Q20.

test	range	mode	median	Q_1	Q_3	I.Q.R.	mean
1	71	21	44	29.5	76.5	47	53.2
2	62	46	67	48.5	81.5	33	66.7

7.3 Groued data, frequencies

Q21.

h	f	$h \times f$
152	4	608
153	2	306
157	3	471
160	3	480
163	6	978
165	3	495
168	3	504

$\therefore$ mean $= \frac{3842}{24} \approx 160$

Q22. $a = 12, b = 4$.

Q23. 6

Q24. 25

Q25. $a = b = 5$, median = 11

Q26. $a = 5, b = 8$

Q27. (1) 3 (3) 5.5 (5) 1

(2) 6 (4) $Q_1 = 5, Q_3 = 6$ (6) 5.44

Q28. (1) 6 (3) 15 (5) 4

(2) 15 (4) $Q_1 = 13, Q_3 = 17$ (6) 13.4

Q29. $a = b = 9$

Q30. (1)

class	f	h (approx.)	$h \times f$
$130 < h \leq 140$	2	135	270
$140 < h \leq 150$	14	145	2030
$150 < h \leq 160$	31	155	4805
$160 < h \leq 170$	33	165	5445
$170 < h \leq 180$	16	175	2800
$180 < h \leq 190$	4	185	740
	100		16090

h	$c.f.$
$h \leq 140$	2
$h \leq 150$	16
$h \leq 160$	47
$h \leq 170$	80
$h \leq 180$	96
$h \leq 190$	100

(2) $160 < h \leq 170$

(3) $150 < Q_1 \leq 160$, $160 < Q_3 \leq 170$

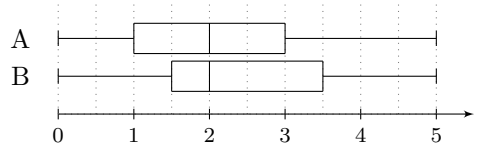
(4) 161 cm

Q31. (1) (6),(7)
(2) (7), 6
(3) (7), 4
(4) (5)
(5) (6)
(6) (a) 7
(b) 0
(c) 4
(d) 1
(7) no
(8) yes
(9) (a) 5
(b) 0
(c) 3
(d) 1

Q32. (1) A: 2.2, B: 2.45
(2) A&B: 2
(3) A: 68, 85, 94, 100, B: 8, 25, 59, 75, 88, 100
(4)

	A	B
median	2	2
Q_1	1	1.5
Q_3	3	3.5

(5)



(6) T - true, N - does not have to be true, F - false

	A	B
(a)	T	T
(b)	N	N
(c)	T	T
(d)	N	T
(e)	T	T
(f)	T	T

Q33. (a) FALSE (b) FALSE (c) FALSE (d) FALSE

Q34. (1) 4 (2) 68 (3) 43-16=27

Q35. (1) 50
(2) 17
(3) 35
(4) 68
(5) $30^{th}: 81-83, 70^{th}: 119-120$
(6) 12 (11-13)
(7) 125
(8) 75
(9) 50%
(10)

time	no of students
$0 < t \leq 20$	2
$20 < t \leq 40$	4
$40 < t \leq 60$	11
$60 < t \leq 80$	18
$80 < t \leq 100$	25
$100 < t \leq 120$	25
$120 < t \leq 140$	18
$140 < t \leq 160$	11
$160 < t \leq 180$	4
$180 < t \leq 200$	2

(11) 100 minutes

Q36. (1) 20-21 minutes
(2) 11
(3) 24
(4) 16
(5) mean ≈ 20

time	no of students
$0 < t \leq 4$	1
$4 < t \leq 8$	2
$8 < t \leq 12$	3
$12 < t \leq 16$	4
$16 < t \leq 20$	5
$20 < t \leq 24$	5
$24 < t \leq 28$	4
$28 < t \leq 32$	3
$32 < t \leq 36$	2
$36 < t \leq 40$	1

Q37. (1) 22hrs

(2) 8 or 7
(3) 15^{th} : $17, 65^{th}$: 24
(4) 27
(5) 16
(6) $c = 16, d = 27$

Q38. (1) 164cm
(2) 173-156=17
(3) 173
(4) 154
(5) $c = 154, d = 173$

7.4 Miscelaneous problems

Q39. (1) mean = 171, range = 12
(2) mean = 172, range = 10 or 11 or 12 or 13 or 14
(3) mean = 171.5, range = 10 or 11 or 12 or 13 or 14

Q40. 960 ml

Q41. (1) $3.25l$
(2) $4.5l$

Q42. (1) 26.40 zł
(2) 24 zł

Q43. $a = 6, b = 7, c = 7, d = 10$

Q44. $a = 4, b = 6, c = 8, d = 8$

Q45. $a = 6, b = c = d = 8, e = f = g = 9$

Q46. $x \in \{8, 9, 10, 11, 12, 13\}, y = 14 - x$

Q47. $x = 3$

Q48. median =9.4, $Q_1 = 9.2, Q_3 = 9.85$,
range = 1.3, $I.Q.R. = 0.65$,
mode = 9.2, mean ≈ 9.468

Q49. 4,6,6,6,7

Q50. 3,4,5,6,6,7

Q51. 2,4,5,5,6,6,7

Q52.

grade	min no	max no
4	1	2
5	0	3
6	3	5
7	2	2

Q53. (1) 76kg
(2) 10-11kg
(3) 30^{th} : $72kg, 70^{th}$: $79kg$
(4) 86
(5) 67
(6) $c = 67, d = 86$
(7) 76kg

Q54. (1) $4000AC$
(2) $1900AC$
(3) 30^{th} : $3400AC, 70^{th}$: $4900AC$
(4) $6500AC$
(5) $2800AC$
(6) $c = 2800, d = 6500$
(7) $4400AC$

Chapter 8
Geometry

8.1 Polygons

Q1. (1) true
(2) true
(3) true
(4) false
(5) true
(6) false
(7) false
(8) false
(9) true (?)
(10) false
(11) true
(12) false
(13) false
(14) true

Q2. $20cm^2$

Q3. $252cm^2$

Q4. $32\sqrt{3} \approx 55.4$

Q5. 64

8.2 Circles

Q6. 67° or 113°

Q7. 156

Q8. $27\sqrt{3}cm$

Q9. $24\sqrt{3} \approx 41.6$

Q10. $256cm^2$

Q11. $36\sqrt{3} \approx 62.4$

Q12. $E\hat{F}G = 80°$, $E\hat{D}G = 100°$ or $E\hat{F}G = 100°$, $E\hat{D}G = 80°$

Q13. 34°

Q14. $A = 58°$, $B = C = 61°$

Q15. $A = 31°$,

$B = 59°$, $C = 90°$

Q16. $36\pi cm^2$

Q17. $78.5°$

Q18. $64\sqrt{2}$

Q19. $24\sqrt{3}cm^2$

Q20. $18cm^2$

Q21. $8\sqrt{2}cm^2$

Q22. $32\sqrt{3}cm^2$

Q23. $96cm^2$

Q24. $27\sqrt{3}$

Q25. $2 : 1$

Q26. $A = 30°$, $B = 60°$, $S = 90°$

Q27. —

8.3 Similarity

Q28. (1) 4.5
(2) —

Q29. $\frac{40}{3}$

Q30. $\frac{2}{3}r\sqrt{6}$

Q31. $\frac{2}{3}r\sqrt{3}$

Q32. —

Q33. 43.56

Q34. $4 : 1$

Q35. 4

Q36. $\frac{16}{3}$

Q37. equal

Q38. $3 : 1$

Q39. (i) 16
(ii) $5 : 3$

Q40. —

8.4 Solid geometry

Q41. $V = \frac{9\sqrt{2}}{2}$, $A = 9 + 9\sqrt{3}$

Q42. $36 + 36\sqrt{7}$

Q43. $224cm^3$

Q44. $1.024\sqrt{6} \approx 2.51(m^3)$

Q45. $141cm$

Q46. $25m40cm$

Q47. $3\sqrt{3} : 1$

Q48. $3\sqrt{3} : 1$

Q49. $1 : 6$

Q50. $\sqrt{3} : 9$

Q51. $5 : 8$

Q52. cube, $3 : 4$

Q53. $V = \frac{2\sqrt{2}}{3}$, $A = 4\sqrt{3}$

Q54. regular octahedron, $1 : 2$

Q55. regular tetrahedron, $1 : 27$

Q56. 30.2%

Q57. 12.3%

Q58. (i) $2\sqrt{2}$
(ii) $\frac{\sqrt{2}}{3}$

8.5 Miscellaneous problems

Q59. 2

Q60. $81(\sqrt{2} - 1)$

Q61. (1) $3\sqrt{2} - 4$
(2) —

Q62. —

Q63. 1.46

Q64. 1.55

Q65. (1) 1
(2) 1.44
(3) 0.95262
(4) $\left(\frac{\sqrt{Rr}}{\sqrt{R}+\sqrt{r}}\right)^2$

Q66. $\frac{2}{3}(2\sqrt{3} - 3)$

Q67. 5

Q68. $\frac{10}{3}$

Q69. 65

Q70. —

Q71. $36\sqrt{3}$

Q72. —

Q73. $5\sqrt{2}$

Q74. $\frac{\sqrt{2}}{2}$

Q75. $\frac{25\pi}{4}(2 - \sqrt{3}) \approx 5.26$

Q76. $154.9m^2$

Q77. 50%

Chapter 9
Numbers II

9.1 Factorials, Pascal's triangle and binomial theorem

Q1. (1) 6 (2) 24 (3) 120 (4) 720 (5) 5040

Q2. (1) 7 (2) 8 (3) 12 (4) 110 (5) 380

Q3. (1) 7 (2) 5 (3) 8

Q4.
(1) 3 (4) 10 (7) 6 (10) 15 (13) 35
(2) 4 (5) 10 (8) 15 (11) 21 (14) 56
(3) 6 (6) 5 (9) 20 (12) 35 (15) 28

Q5. (1) $\frac{n(n+1)}{2}$ (2) $\frac{3n(3n-1)}{2}$ (3) $\frac{n(n-1)}{2}$ (4) $\frac{n(n+1)}{2}$ (5) $n(2n-1)$ (6) $\frac{(n+2)(n+1)n}{6}$

Q6.
(1) $a^2 + 2ab + 2ac + b^2 + 2bc + c^2$
(2) $a^4 - 2a^2b + 4a^2c + b^2 - 4bc + 4c^2$
(3) $a^4 + 2a^3b + 3a^2b^2 + 2ab^3 + b^4$
(4) $a^4 + 2a^3 - a^2 - 2a + 1$
(5) $25x^2y^2 + 20x^2y + 4x^2 - 30xy^2 - 12xy + 9y^2$
(6) $9a^2b^2 + 12a^2bc + 4a^2c^2 - 6ab^2c - 4abc^2 + b^2c^2$

(7) $4a^4b^2 + 12a^3b^3 + 9a^2b^4 + 4a^2b + 6ab^2 + 1$
(8) $9s^2t^2 - 12s^2t + 4s^2 - 12st^3 + 8st^2 + 4t^4$
(9) $9 + 4\sqrt{2} + 4\sqrt{3} + 2\sqrt{6}$
(10) $9 + 4\sqrt{2} - 4\sqrt{3} - 2\sqrt{6}$
(11) $10 + 2\sqrt{6} + 2\sqrt{10} + 2\sqrt{15}$
(12) $11 + 6\sqrt{2} - 4\sqrt{3} - 2\sqrt{6}$

Q7. (1) $a^3 - 3a^2c + 3ac^2 - c^3$
(2) $a^6 + 6a^4b + 12a^2b^2 + 8b^3$
(3) $a^6 - 3a^5b + 3a^4b^2 - a^3b^3$
(4) $a^6 + 6a^5 + 12a^4 + 8a^3$
(5) $8x^6y^3 + 36x^5y^4 + 54x^4y^5 + 27x^3y^6$
(6) $27x^6y^3 - 54x^5y^4 + 36x^4y^5 - 8x^3y^6$
(7) $54 + 30\sqrt{3}$
(8) $11\sqrt{2} + 9\sqrt{3}$
(9) $9\sqrt{3} - 11\sqrt{2}$
(10) $132\sqrt{3} - 162\sqrt{2}$
(11) $21\sqrt{3} + 15\sqrt{6}$
(12) $12\sqrt{6} - 20\sqrt{2}$

Q8. (1) $a^4 - 4a^3c + 6a^2c^2 - 4ac^3 + c^4$
(2) $x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$
(3) $193 - 132\sqrt{2}$
(4) $a^5 - 5a^4c + 10a^3c^2 - 10a^2c^3 + 5ac^4 - c^5$
(5) $x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$
(6) $843 - 589\sqrt{2}$
(7) $485 + 198\sqrt{6}$
(8) $64a^6b^6 - 576a^6b^5 + 2160a^6b^4 - 4320a^6b^3 + 4860a^6b^2 - 2916a^6b + 729a^6$

Q9. (1) 63
(2) 0
(3) 30
(4) 58
(5) $\frac{15}{16}$
(6) 28
(7) 11
(8) 24

Q10. (1) $\sum_{r=1}^{20} r$
(2) $\sum_{r=1}^{22} 2r$
(3) $\sum_{r=1}^{n} r$
(4) $\sum_{r=1}^{26}(2r-1)$
(5) $\sum_{r=0}^{10} 2^r$
(6) $\sum_{r=1}^{8} 3^{2-r}$
(7) $\sum_{r=1}^{40}(5-2r)$
(8) $\sum_{r=1}^{20}(4r-1)$
(9) $\sum_{r=1}^{16}(13-3r)$
(10) $\sum_{r=1}^{17}(\frac{1+r}{3})$
(11) $\sum_{r=0}^{11}(-2)^r$
(12) $\sum_{r=1}^{8}(3^r(-1)^{r+1})$
(13) $\sum_{r=0}^{12}(-2)^{-r}$
(14) $\sum_{r=1}^{20}(\frac{(-1)^{r+1}}{r})$
(15) $\sum_{r=1}^{25} r^2$
(16) $\sum_{r=1}^{11}(r^3(-1)^{r+1})$
(17) $\sum_{r=1}^{99} \frac{r}{r+1}$
(18) $\sum_{r=1}^{26} \frac{(-1)^{r+1}}{2r}$
(19) $\sum_{r=1}^{21}(r(r+1))$
(20) $\sum_{r=1}^{18} \frac{2r}{(2r-1)(2r+1)}$

Q11. -4320

Q12. -20

Q13. $\frac{560}{3}$

Q14. 160

Q15. 840

Q16. -489888

Q17. 672

Q18. -448

Q19. -405

Q20. 3840

Q21. $2048x^{11} - 16896x^{10} + +63360x^9$

Q22. $-1152x^2 + 1152x - 512$

Q23. $8 - 20x + 18x^2$

Q24. $-1 + 3x - 10x^3$

Q25. $-1 - 5x + 40x^3$

Q26. $2 + 12x - 21x^2$

Q27. 360

Q28. $-\frac{1567}{9}$

Q29. (1) $1 - 2x - 6x^2 + \ldots$
(2) $4 - 28x + 65x^2 + \ldots$
(3) $-4 - 31x - 88x^2 + \ldots$
(4) $1 + x - \frac{5}{4}x^2 + \ldots$

Q30. (1) $-8x^7 - 20x^6 - 6x^5 + \ldots$
(2) $16x^6 + 32x^5 - 40x^4 + \ldots$
(3) $16x^5 - 32x^4 - 104x^3 \ldots$
(4) $\frac{1}{64}x^8 + \frac{5}{32}x^7 + \frac{37}{64}x^6 + \ldots$

Q31. 2

Q32. ± 3

Q33. $-\sqrt[3]{2}$

9.2 Logarithms

Algebra of logarithms

Q34. (1) 3 (2) $\frac{1}{4}$ (3) $\frac{\sqrt{2}}{2}$ (4) $\frac{25}{2}$ (5) $4^6 = 2^{12}$ (6) 11×3^n

Q35. (1) 3 (2) $\frac{3}{2}$ (3) $\frac{2}{3}$ (4) $\frac{3}{2}$ (5) $\frac{1}{4}$ (6) $-\frac{1}{2}$ (7) $-\frac{3}{2}$ (8) -3 (9) $-\frac{7}{4}$ (10) 3.5 (11) 2.5 (12) $\frac{5}{4}$ (13) -5 (14) 5 (15) -2 (16) $-\frac{3}{2}$ (17) $\frac{3}{5}$ (18) $\frac{1}{2}$ (19) $-\frac{8}{3}$ (20) $\frac{5}{6}$ (21) $\frac{3}{4}$ (22) $-\frac{7}{4}$ (23) $\frac{1}{12500}$ (24) $\frac{2}{\sqrt{3}}$ (25) $2 \times 3^{\frac{1}{4}}$

Q36. (1) $2b$ (2) $a + b$ (3) $2a + b$ (4) $3 + b$ (5) $2 + a + b$ (6) $3 + a + 3b$ (7) $3a$ (8) $3 + 2a$

Q37. (1) $a + 2b$ (2) $2a + 4b$ (3) $\frac{1}{2}a + b$ (4) $a + 2b$ (5) $a + 6b$ (6) $6b$ (7) $\frac{1}{2}a + 4b$ (8) a

Q38. (1) $-a$ (2) $2a$ (3) $\frac{1}{a}$ (4) $1 + \frac{3}{a}$ (5) $\frac{1}{2}a$ (6) $\frac{1}{3}a$ (7) $\frac{2+5a}{3}$ (8) $\frac{5}{2a}$

Q39. (1) $-a - 3b$ (2) $\frac{1}{2}a - b$ (3) $a - \frac{1}{2}b$ (4) $1 - 4a - \frac{3}{2}b$

Q40. (1) $a - 2b - \frac{1}{2}c$ (2) $3a + \frac{1}{2}b - 2c)$ (3) $2.5a - 1.5b - c$ (4) $-4a - 2.5b - \frac{4}{3}c$

Q41. (1) $a = 2, b = 2$ (2) $a = 2, b = 1$ (3) $a = 4, b = 1$ (4) $a = 3, b = 3$ (5) $a = 3, b = 1$ (6) $a = 4, b = 2$

Q42. (1) $\frac{3}{4}(2a - 1)$ (2) $\frac{2a+3}{4a-2}$ (3) $\frac{3}{2a-1}$ (4) $\frac{14-8a}{3a-3}$

Q43. (1) $\sqrt{3}$ (2) 25 (3) 8 (4) $3^{(\frac{4}{3})}$ (5) $2\sqrt{2}$ (6) $\frac{1}{9}$ (7) $\frac{1}{4}$ (8) 8

Q44. (1) $2a$ (2) $\frac{a}{2}$ (3) $32a$ (4) $8a^2$ (5) $\frac{1}{3}b^3$ (6) a^2 (7) a^3 (8) $\frac{1}{b^2}$ (9) c^2 (10) a^4 (11) $\frac{1}{a^4}$ (12) b^3 (13) $\frac{1}{16}a^2$ (14) $\frac{1}{c}$ (15) $\frac{16}{a}$ (16) $\frac{729}{b^2}$ (17) c^4 (18) $\frac{1}{8}a^6$ (19) $\frac{256}{a^{12}}$ (20) $\frac{b^6}{243}$ (21) $\frac{1}{3}$ (22) $\frac{25}{c^5}$ (23) ac (24) $\frac{a^2b^4}{18}$

Logarithmic equations

Q45. (1) $\frac{27}{64}$ (2) 27 (3) 8 (4) 9 (5) $5^5 = 3125$ (6) $\sqrt[3]{2}$ (7) $\sqrt[3]{2}$ (8) $\frac{1}{2}$ (9) no solutions (10) $x = \frac{64}{\pi^3}$ (11) $x = \frac{1}{8}$ (12) $x = \frac{1}{5}$ or $x = 5$

Q46. (1) no solutions (2) $\frac{11}{4}$ (3) 7 (4) $\frac{17}{4}$ (5) $-\frac{1}{2}$ (6) 9 (7) 0.1 (8) no solutions

Q47. (1) $\frac{1}{2}$ (2) 27 (3) $\sqrt{3}$ (4) 7

Aplications

Q48. (i) 5 milion
(ii) $\frac{1}{6} \approx 0.167$
(iii) 39.9 years

Q49. (i) 15 litres
(ii) 34.3 minutes
(iii) 228 minutes

Q50. (i) 0.242
(ii) 965 thousands
(iii) 29

9.3 Absolute value equations and inequalities

Q51. (1) $-\frac{1}{2}, \frac{1}{2}$ (2) $\frac{1}{2}, 1$ (3) $0, 2$ (4) $-\frac{2}{3}, 2$ (5) $\frac{1}{6}, \frac{3}{2}$ (6) $\frac{2}{5}, 4$ (7) $-4, 0$ (8) $2, 4\frac{2}{3}$ (9) $-7, -2.5$

Q52. (1) $-1.5, 3$ (2) $-\frac{4}{3}, 8$ (3) $-1, 3$ (4) $1, 7$ (5) $0, 2.4$ (6) $-\frac{3}{2} \leq x \leq \frac{1}{2}$ (7) $-\frac{1}{4}$ (8) $-1, \frac{9}{7}$

Q53. (1) $x \neq -2$ (2) $x \in]-\infty, 1[\cup]2, +\infty[$ (3) $x \in]-\infty, -2] \cup [\frac{4}{3}, +\infty[$ (4) $x \in \mathbb{R}$ (5) $x \in]-\frac{9}{4}, -\frac{1}{4}[$ (6) $x \in \emptyset$ (7) $x \in [-\frac{5}{2}, -\frac{1}{2}]$ (8) $x \in \emptyset$ (9) $x \in \mathbb{R}$ (10) $x \in]-\frac{1}{2}, \frac{9}{2}[$ (11) $x \in [-\frac{3}{2}, \frac{1}{2}]$ (12) $x \in]-\infty, -\frac{3}{2}] \cup [\frac{9}{2}, +\infty[$ (13) $x \in]-\infty, \frac{2}{3}[\cup]2, +\infty[$ (14) $x \in \emptyset$ (15) $x = \frac{5}{2}$ (16) $x \in \mathbb{R}$ (17) $x \in]-\infty, 1] \cup [4, +\infty[$ (18) $x \in [\frac{4}{3}, 2]$ (19) $x \in]-\infty, -\frac{7}{2}[\cup]\frac{1}{2}, +\infty[$ (20) $x \in]-\frac{4}{3}, \frac{8}{3}[$

Q54. (1) $x < 0$ or $x > 2$ (2) $-2 < x < 0$ (3) $-\frac{2}{3} \leq x \leq 0$ (4) $-\frac{1}{5} \leq x \leq 5$ (5) $x < -4$ or $-4 < x < -\frac{2}{5}$ or $x > 2$ (6) $-\frac{2}{5} < x < 2$, $x \neq \frac{1}{2}$ (7) $-\frac{3}{2} \leq x \leq -\frac{1}{6}$, $x \neq -\frac{1}{2}$ (8) $0 < x < 4$ (9) $x < -1$ or $-1 < x \leq -\frac{1}{8}$ or $x \geq 2.5$ (10) $\frac{4}{5} \leq x \leq \frac{16}{3}$

Q55. (1) $1.6 \leq x \leq 4$ (2) $x < 1$ or $x > 3.5$ (3) $-\frac{7}{2} < x < -1$ (4) $x \leq -1$ or $x \geq 3$

9.4 Complex numbers

Q56. (1) $2 + 11i$ (2) $11 + 13i$ (3) 20 (4) $13i$ (5) $-9 + 2i$ (6) $19 - 9i$ (7) $9 + 12i$ (8) $8 + 0.25i$ (9) $-6 - 8i$ (10) $-10 + 10i$ (11) 13 (12) $4 - 7i$

Q57. (1) $\frac{2}{5} - \frac{1}{5}i$ (2) $\frac{1}{29} - \frac{17}{29}i$ (3) $\frac{3}{5} + \frac{4}{5}i$ (4) $-\frac{12}{13} + \frac{5}{13}i$ (5) $1.4 + 1.2i$ (6) $\frac{1}{13} + \frac{21}{13}i$ (7) $-1 - \frac{4}{3}i$ (8) $\frac{-8}{41} + \frac{31}{41}i$ (9) $\frac{6}{25} - \frac{8}{25}i$ (10) $\frac{7}{10} + \frac{1}{10}i$ (11) $-\frac{5}{13} + \frac{12}{13}i$ (12) $\frac{8}{5} - \frac{1}{5}i$

Q58. (1) $2 - 11i$ (2) $-11 - 2i$ (3) -4 (4) -64 (5) $-8 - 8i\sqrt{3}$ (6) $-16 + 16i\sqrt{2}$ (7) $-16 + 16i\sqrt{3}$ (8) 1

Q59. (1) $a = 2$, $b = -9$ (2) $a = 3$, $b = \pm 2$ (3) $a = -2$, $b = 1$ (4) $a = 3$, $b = -4$ (5) $a = \pm 2$, $b = \pm 1$ (6) $a = 0$, $b = -2$ (7) $a = 2$, $b = -1$ (8) $a = \pm 3$, $b = 2$

Q60. (1) $-1 - 2i$ (2) $3i$ (3) $1 + 3i$ (4) $1 - 2i$ (5) $3 + i$ (6) $1 + 2i$ (7) $1 - 3i$ (8) $3 + 2i$

9.5 Mathematical induction

Q61. —

Chapter 10
Quadratics, circles and polynomials

10.1 Vieta's formulae for quadratics

Q1. (1) 2; different signs (2) 2; both negative (3) 0 (4) 1; negative (5) 2; both negative (6) 2; different signs (7) 1; positive (8) 2; both positive (9) 0 (10) 2; both positive (11) 2; both positive (12) 2; different signs

(13) 2; both negative
(14) 1; positive
(15) 2; both positive
(16) 2; different signs
(17) 2; both positive
(18) 2; both negative
(19) 2; different signs
(20) 2; both negative

Q2. $0 < m < 1$

Q3. $-2 < m < 0$

Q4. no such m

Q5. $-2.5 < m \leq -0.5$ or $m \geq 0.5$

Q6. $\frac{2}{9} < m < \frac{1}{3}$

Q7. $\frac{1}{3} < m \leq \frac{2+\sqrt{6}}{4}$

Q8. $0 \leq m \leq \frac{-7+5\sqrt{2}}{2}$

Q9. $-2 < m < -\frac{1}{2}$

Q10. $m \leq -10 - 6\sqrt{3}$ or $m > 2$

Q11. $\frac{18-8\sqrt{3}}{11} \leq m < \frac{2}{3}$

Q12. $\frac{4}{3} \leq m < \frac{3}{2}$

Q13. (i) $m < 0$ or $m \geq \frac{7+4\sqrt{2}}{3}$
(ii) $-1 < m \leq \frac{7-4\sqrt{2}}{3}$, $x \neq \frac{1}{3}$

Q14. (i) $m < -2$ or $m \geq 10 + 6\sqrt{3}$
(ii) $-1 < m \leq 10 - 6\sqrt{3}$, and $m \neq -\frac{1}{2}$

10.2 Algebraic fractions

Q15. (1) $\frac{x+2}{2x+1}$ (2) $\frac{2x-3}{x+2}$ (3) $\frac{3x-2}{2x+5}$ (4) $\frac{2x+1}{2x+3}$ (5) $\frac{3x-2}{2x+3}$ (6) $\frac{3x-2}{2x-3}$ (7) $\frac{5x+1}{5x-1}$ (8) $\frac{2x+5}{5x+2}$ (9) $\frac{8x+2}{x-2}$ (10) $\frac{6x-5}{2x+1}$ (11) $\frac{2x-5}{5x+2}$ (12) $\frac{-3x+4}{2x+1}$

Q16. (1) $-\frac{4x+8}{3x-2}$ (2) $-\frac{3}{2}$ (3) $\frac{2}{3}$ (4) $\frac{24x^2-22x+4}{5}$ (5) $\frac{6}{6x^2+x-1}$ (6) $\frac{2x^2-3x+1}{2x^2+3x+1}$ (7) $\frac{-4x+6}{6x+3}$ (8) $\frac{2x^2+7x+6}{2x^2-7x+6}$ (9) $\frac{6x^2-x-1}{6x^2+x-1}$

Q17. (1) $\frac{5x+12}{(x+2)(x+3)}$ (2) $\frac{-x}{(2x-3)(x-2)}$ (3) $\frac{24x}{(3x-2)(3x+2)}$ (4) $\frac{-12}{(2x+1)(2x-3)}$ (5) $\frac{4x+3}{(3x-4)(2x-1)}$ (6) $\frac{14x+16}{(3x-2)(2x+5)}$ (7) $\frac{17x+7}{(5x+1)(x+2)}$ (8) $\frac{8x+14}{(2x+5)(2x-1)}$ (9) $\frac{-18x+5}{(6x+2)(3-2x)}$ (10) $\frac{3x-1}{(6x-5)(3x-2)}$ (11) $\frac{4x+19}{(2x-5)(5x+2)}$ (12) $\frac{-6x+7}{(4-3x)(2x-3)}$

Q18. (1) $\frac{7x^2+x-5}{(2x+1)(x-2)(x+3)}$ (2) $\frac{x^2+7x+13}{(x+2)(x+3)(x-2)}$ (3) $\frac{-36x+5}{(2x+5)(2x-5)(3x-2)}$ (4) $\frac{-14x-8}{(2x+3)(3x-2)(2x-3)}$ (5) $\frac{14x^2-3x+3}{(2x+3)(2x-3)(2x+1)}$ (6) $\frac{21x^2-20x-1}{(2x-3)(5x-1)(2x+5)}$ (7) $\frac{10x^2+29x-1}{(5x-1)(3x+2)(x+2)}$ (8) $\frac{11x^2+7x-4}{(5x+2)(3x)(2x-1)}$ (9) $\frac{20x^2-2x-4}{(x-2)(3x-2)(2x-3)}$ (10) $\frac{12x^2-6x-2}{(2x+1)(3x+1)(3x-2)}$ (11) $\frac{-19x^2-25x}{(5x+2)(3x)(5x+2)}$ (12) $\frac{-15x^2+18x+10}{(2x+1)(5x+2)(2x-3)}$

10.3 Equation of a circle

Q19. (1) centre: $(2, 1)$, radius $=5$
(2) centre: $(0, -2)$, radius $=4$
(3) centre: $(3, 0)$, radius $=3$
(4) centre: $(4, -4)$, radius $=4$
(5) centre: $(1.5, -2.5)$, radius $=3$
(6) centre: $(-5, 2)$, radius $=7$
(7) centre: $(-3, 1)$, radius $=3\sqrt{2}$
(8) centre: $(1, 1.5)$, radius $=4\sqrt{3}$
(9) centre: $(-3, 1.5)$, radius $=3\sqrt{2}$
(10) centre: $(-1.5, -0.5)$, radius $=2\sqrt{2}$
(11) centre: $(2, 0.5)$, radius $=5\sqrt{3}$
(12) centre: $(0.5, -3)$, radius $=2\sqrt{10}$

Q20. (1) $(5, 5)$, $(-2, 4)$
(2) $(4, -5)$, $(5, -2)$
(3) $(4, 7)$, $(8, -5)$
(4) $(6, 5)$, $(-2, 3)$
(5) $(-3.5, 2.5)$, $(0.5, -9.5)$
(6) $(3, 1)$, $(-7.15, -5.77)$
(7) $(-10, -3)$, $(-2, -7)$
(8) $(8, 0.5)$, $(6, -3.5)$
(9) $(2, 6.5)$, $(-10, 0.5)$
(10) $(5.5, -6.5)$, $(0.5, -9.5)$
(11) $(9, -3.5)$, $(1, 8.5)$
(12) $(9.5, -1)$, $(7.5, -9)$
(13) $(2, 1)$
(14) $(1, 1)$
(15) $(-1, -3)$
(16) $(-2, 1)$

Q21. (1) $(8, 1)$, $(7, 2)$
(2) $(6, 2)$, $(9, -2)$
(3) $(-2, -2)$, $(6, 2)$
(4) $(-2, -2)$, $(0, 4)$

Q22. $(x-8)^2 + (y-1)^2 = 25$

Q23. $(x+2.5)^2 + (y+0.5)^2 = 32.5$

Q24. $(x-3)^2+(y-\frac{3}{4})^2=\frac{625}{16}$

Q25. $(x-4)^2+(y-10)^2=180$

Q26. $(-1.5,4)$ or $(-0.5,2)$

Q27. $(-5.5,-3.5)$ or $(3.5,-0.5)$

Q28. $\sqrt{13}$ or $5\sqrt{13}$

Q29. $b=2$ or $b=-4$, $r=5\sqrt{2}$

Q30. $a=2$ or $a=-10$, $r=5\sqrt{5}$

Q31. 2 or $\frac{1}{2}$

Q32. -3 or $-\frac{1}{3}$

Q33. -6 or 14

Q34. (1) $(-3,-6)$, $(3,-6)$, $(-4,1)$, $(4,1)$

(2) $(0,-7)$, $(-3,2)$, $(3,2)$

(3) $(0,3)$, $(3,-6)$, $(4,-5)$, $(5,-2)$

Q35. $(x-7)^2+(y+1)^2=50$

10.4 Polynomials

Q36. (1) $x\in]1,+\infty[$
(2) $x\in]-\infty,1]$
(3) $x\in]-\infty,0[$
(4) $x\in\{-1\}\cup[0,+\infty[$
(5) $x\in]-\infty,-1]\cup[0,3]$
(6) $x\in]-1,1[\cup]1,+\infty[$
(7) $x\in]-\infty,-3]\cup[-1,1]$
(8) $x\in]\frac{1}{3},\frac{1}{2}[\cup]\frac{1}{2},+\infty[$
(9) $x\in]-\infty,-\frac{1}{3}[$
(10) $x\in]-\infty,-\frac{2}{3}[\cup]-\frac{2}{3},\frac{1}{3}[$
(11) $x\in[-1,\frac{1}{2}]\cup[2,+\infty[$
(12) $x\in]-\infty,\frac{2}{3}]\cup\{\frac{3}{2}\}$
(13) $x\in]\frac{2}{3},\frac{3}{2}[\cup]\frac{3}{2},+\infty[$
(14) $x\in\{\frac{3}{5}\}\cup[\frac{3}{4},+\infty[$
(15) $x\in]-\infty,-2[\cup]\frac{1}{4},3[$
(16) $x\in]-\infty,-5]\cup\{\frac{7}{5}\}$
(17) $x\in\{\frac{7}{3}\}\cup[\frac{12}{5},+\infty[$
(18) $x\in]-\infty,-\frac{3}{2}[\cup]-\frac{2}{3},\frac{3}{2}[$

Q37. (1) $x\in]\frac{1}{5},+\infty[$
(2) $x\in]-\infty,\frac{2}{5}]$
(3) $x\in]-\infty,0[$
(4) $x\in\{-2\}\cup[0,+\infty[$
(5) $x\in]-\infty,-1]\cup[0,2]$
(6) $x\in]-1,0[\cup]1,+\infty[$
(7) $x\in]-\infty,-3]\cup[-1,0]$
(8) $x\in]0,\frac{1}{2}[\cup]\frac{1}{2},+\infty[$
(9) $x\in]-\frac{1}{3},0[\cup]\frac{2}{3},+\infty[$
(10) $x\in]-\infty,-\frac{2}{3}[\cup]0,\frac{1}{3}[$
(11) $x\in]-\infty,-1]\cup[0,\frac{1}{2}]$
(12) $x\in]-\infty,0]\cup[\frac{2}{3},\frac{3}{2}]$
(13) $x\in]0,\frac{3}{2}[\cup]\frac{3}{2},+\infty[$
(14) $x\in[0,\frac{3}{5}]\cup[\frac{3}{4},+\infty[$
(15) $x\in]-\infty,-2[\cup]0,\frac{1}{4}[$
(16) $x\in]-\infty,-5]\cup[0,\frac{7}{5}]$
(17) $x\in[-\frac{10}{3},0]\cup[\frac{7}{3},+\infty[$
(18) $x\in]-\infty,-\frac{3}{2}[\cup]0,\frac{3}{2}[$

Q38. (1) $x\in]-3,2[\cup]3,+\infty[$
(2) $x\in]-\infty,-2]\cup[1,2]$
(3) $x\in]-\frac{1}{2},\frac{1}{4}[\cup]\frac{1}{2},+\infty[$
(4) $x\in[-\frac{3}{2},-\frac{1}{2}]\cup[\frac{3}{2},+\infty[$
(5) $x\in]-\infty,-\frac{5}{2}]\cup[1,\frac{5}{2}]$
(6) $x\in]-\frac{5}{3},\frac{3}{2}[\cup]\frac{5}{3},+\infty[$
(7) $x\in[-4,-2]\cup[4,+\infty[$
(8) $x\in]-\infty,-\frac{2}{3}[\cup]\frac{1}{2},\frac{2}{3}[$
(9) $x\in]-\infty,-\frac{2}{3}[\cup]\frac{1}{2},\frac{2}{3}[$
(10) $x\in]-\infty,-\frac{1}{3}[$
(11) $x\in[-\frac{1}{2},\frac{2}{5}]\cup[\frac{1}{2},+\infty[$
(12) $x\in\{-\frac{3}{2}\}\cup[\frac{3}{2},+\infty[$
(13) $x\in]-\frac{3}{2},\frac{3}{2}[\cup]\frac{3}{2},+\infty[$
(14) $x\in]-\infty,\frac{3}{4}]$
(15) $x\in]-\frac{1}{4},\frac{1}{5}[\cup]\frac{1}{4},+\infty[$
(16) $x\in]-\infty,-\frac{7}{5}]\cup[\frac{4}{3},\frac{7}{5}]$
(17) $x\in[-\frac{7}{3},\frac{3}{2}]\cup[\frac{7}{3},+\infty[$
(18) $x\in]-\infty,-\frac{3}{2}[$

Q39. (1) quotient: x^3+3x^2-2x-1, remainder: 1
(2) quotient: x^3-x^2-2x+3, remainder: -1
(3) quotient: $2x^4-2x^3-3x^2+6x+3$, remainder: 6
(4) quotient: $3x^4-x^3-2x+3$, remainder: -5
(5) quotient: $-x^4+2x^2-2x+1$, remainder: -7
(6) quotient: $-2x^4+3x^2-2x+1$, remainder: -4
(7) quotient: x^3-x^2-2x+3, remainder: $2x-1$
(8) quotient: x^3+2x^2-2x+3, remainder: $2x+3$
(9) quotient: $x^4+x^3-x^2-2x+3$, remainder: -1
(10) quotient: $2x^4+5x^3-x^2-2x+3$, remainder: $2x-3$
(11) quotient: $3x^4+x^3+2x^2-2x+3$, remainder: $3x-2$
(12) quotient: $2x^4+x^3-2x^2+3$, remainder: $4x$

Q40.

(1) $x^3+3x^2-2x-1+\frac{2x+1}{3x^2-2}$
(2) $4x^2-2x+3+\frac{3x-1}{x^2+x-2}$
(3) $2x^3-x+3+\frac{x+4}{x^2-2x-3}$
(4) $2x^3-3x^2+x+\frac{3x-5}{3x^2+x-2}$
(5) $3x^3-5x^2+4x-2+\frac{-x+4}{x^2-3x+2}$
(6) $2x^3-4x^2+3x+1+\frac{4}{x^2-2x+1}$
(7) $x^3-6x^2-x+1+\frac{5x+6}{2x^2-3x-2}$
(8) $2x^3-x^2-x-1+\frac{-8}{-3x^2+2x-1}$
(9) $x^3+2x^2+3x+4+\frac{5x+6}{x^2-3x+5}$
(10) $4x^3-3x^2+2x-1+\frac{-x+5}{x^2-2x+3}$
(11) $2x^3-x^2+3+\frac{3x+3}{2x^2+2x-1}$
(12) $x^3+2+\frac{2x-1}{x^2+x-1}$

Q41. (1) -3 (2) 3 (3) 12 (4) $\frac{7}{2}$ (5) $\frac{5}{3}$ (6) $\frac{281}{8}$ (7) $-\frac{4}{25}$ (8) $-\frac{8}{9}$ (9) $-\frac{3}{4}$ (10) $\frac{9}{4}$ (11) $-\frac{11}{32}$ (12) $\frac{40}{9}$

Q42. (i) $-\frac{1}{2}$ (ii) $(x+1)(2x+1)(x-2)$

Q43. (i) $-\frac{1}{3}$ (ii) $(x+2)(x-3)(3x+1)$

Q44. (i) $\frac{3}{2}$ (ii) $(2x-3)^2(2x-3)$

Q45. (i) 2 (ii) $(2x-5)^2(x-2)$

Q46. (i) -4 (ii) $(x+4)(x-5)(3x-4)$

Q47. (i) 2 (ii) $(3x+5)^2(x-2)$

Q48. (i) $-\frac{3}{4}$ (ii) $(4x-1)(4x+3)(2x-1)$

Q49. (i) $-\frac{4}{3}$ (ii) $(3x-2)(3x+1)(3x+4)$

Q50. (ii) $(0,5)$, $(-4,2)$, $(-5,5)$

Q51. (i) $(0,10)$, $(6,4)$ (iii) $(4,2)$ (iv) they are tangent

Q52. (i) $y=-\frac{3}{2}x+9$ (ii) $(6,0)$, $(1,7.5)$ (iv) $(2,8)$, $(-1,3.5)$

Q53. $(-2,5)$, $(1,0.5)$, $(2,1)$

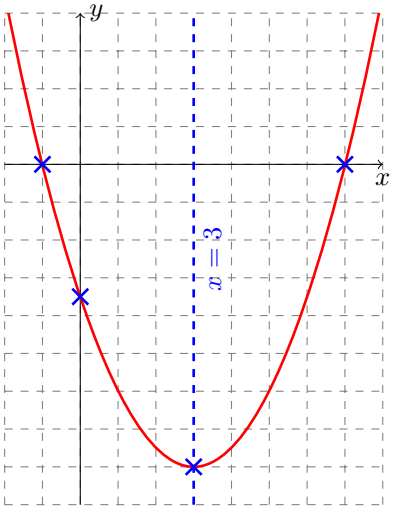
Q54. (ii) $(3,1)$, $(5,3)$ (iii) $(x-2)^2+(y-4)^2=10$ (iv) $(1,7)$

10.5 Chapter review

Q1. (i) $f(x)=\frac{1}{2}(x-3)^2-8$, vertex is $(3,-8)$

(ii) $\frac{1}{2}(x-7)(x+1)=0$, so $x=-1$ or $x=7$

(iii)



Q2. *Answers below are examples only. There are a lot more possible answers.*

(1) $x^2+3<0$ (2) $x^2+3>0$ (3) $(x-2)^2>0$ (4) $(x-5)^2\le 0$ (5) $(x+3)(x-5)<0$ (6) $(x-4)(x-6)\ge 0$

Q3. (1) $a>0$ (2) $b<0$, $c<0$ (3) $a>0$, $b<0$

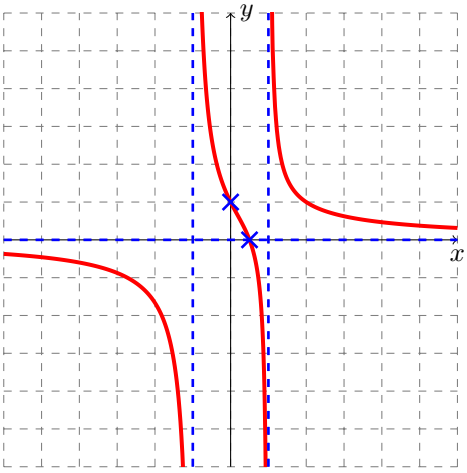
Q4. (1) $2<m<3$

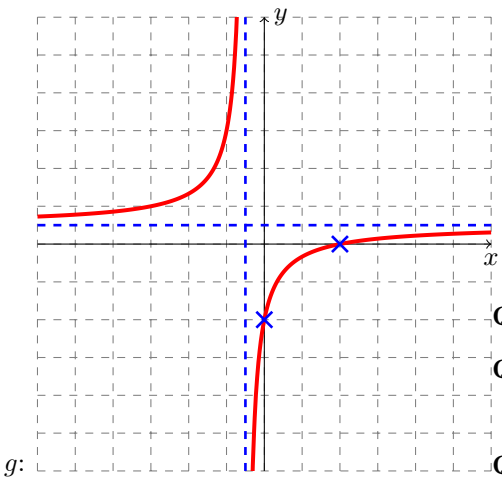
(2) $-1<m<2$

Q5. (i) —

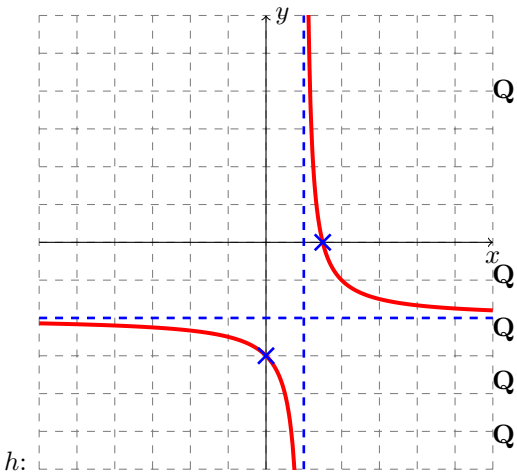
(ii) $g(x)=\frac{x-2}{2x+1}$, $h(x)=\frac{2x-3}{1-x}$

(iii) f:





g:



h:

Q6. (1) $x < -1$ or $0 < x < 2$ or $x > 2$

(2) $-1 < x < 2$, $x \neq 0$

(3) $x \geq 1$

(4) $x \geq 3$ or $x = 1$ or $x = 2$

(5) $1 \leq x \leq 3$

(6) $x < 0$ or $1 < x < 3$

(7) $1 < x < 3$

(8) $-\frac{4}{5} \leq x \leq 0$ or $x \geq \frac{4}{5}$

(9) $x \geq 1$

(10) $-\sqrt{2} < x < \frac{2}{3}$ or $x > \sqrt{2}$

Q7. 6 and 8

Q8. 100 m (parallel to the additional fencing) and 150

Q9. width is 9 cm, height is 10 cm

Q10. yes

Q11. (i) $(4, -4)$, $\sqrt{10}$

(ii) $P = 4\sqrt{10}$, $A = 8$

Q12. (1) $0 \leq x \leq 3$

(2) $\frac{3-3\sqrt{3}}{2} \leq x < 1$ or $4 < x \leq \frac{3+3\sqrt{3}}{2}$

Q13. (1) 1.5, $-2 \pm i\sqrt{3}$

(2) $-\frac{2}{3}$, $3 \pm 2i$

(3) -2.5, $\pm i\sqrt{3}$

(4) $\frac{1}{5}$, $\pm\frac{1}{2}i$

Q14. $f(x) = (600 - 2x)(500 + 40x)$; 420

Q15. $\frac{5}{2} - \sqrt{6} < m < \frac{5}{2} + \sqrt{6}$

Q16. 123 meters

Q17. (i) $(2, 3)$

(ii) 5

(iii) $(-2, 0)$, $(6, 0)$

(iv) inside

(v) $x - 7y + 2 = 0$

(vi) $7x + y - 42 = 0$

(vii) $(5, 7)$

Q18. (i) $(-3, 11.5)$, $(5, 3.5)$

(ii) $(0, 1)$, $(6, 7)$